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OSCAR FABER

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SIMPLE EXAMPLES OF REINFORCED CONCRETE DESIGN

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SECOND EDITION

OXFORD UNIVERSITY PRESS
LONDON: HUMPHREY MILFORD
1929

Oxford University Press
London Edinburgh Glasgow Leipzig
New York Toronto Melbourne Cape Town
Bombay Calcutta Madras Shanghai
Humphrey Milford Publisher to the UNIVERSITY

FIRST PRINTED IN 1924
SECOND EDITION, 1929
PRINTED IN GREAT BRITAIN

PREFACE

THESE examples are intended to assist a student or a beginner in reinforced concrete design to produce reasonable designs for simple structures.

They are intended to be simple practical applications of the principles enunciated in *Reinforced Concrete Simply Explained,* which first appeared in the form of articles in *Concrete and Constructional Engineering* and was subsequently reprinted and published in book form by the Oxford University Press.

They are not intended, therefore, to represent the author's views on the most finished or economical design, and it is to be clearly understood that the author would in many cases allow an experienced designer to adopt somewhat smaller sections or reinforcements. But this needs a thorough understanding of many *finesses* in design, coupled with a good judgment of whether the conditions under which the *execution* of the work will be undertaken are likely to be good or bad—the workmanship, quality of materials, weather, supervision, etc.—and the knowledge and experience of these matters cannot fairly be expected in a beginner in design.

What *is* aimed at is to present, in a simple form, designs of simple structures which under average conditions of workmanship, etc., ought to give satisfactory results. Obviously a structure of any magnitude or importance ought always to be designed and constructed under the supervision of a really competent and experienced man, whose fees are saved many times over by the greater economy of his design and the certainty that the result will be a lasting success. The simplest and clearest language will be used at the risk of wearying readers with previous knowledge.

OSCAR FABER.

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SIMPLE EXAMPLES OF REINFORCED CONCRETE DESIGN

EXAMPLE I

CIRCULAR WATER TANK

Circular water tank, 10 ft. diam., 10 ft. high, resting on the soil.

DESIGN.—If a flexible cup of the same shape as our tank, namely, cylindrical top and flat bottom, is made of waterproof fabric of any kind and stood on a flat board and filled with water, it will be found that the water will exert a pressure against the sides which will cause them to take up a circular form.

Such a cup will be found to be quite a satisfactory container for water on a small scale, and obviously fulfils the functions, as regards structural resistance, which our tank has to resist.

Now, it is clear that the cup of flexible fabric cannot anywhere be exposed to bending or compression stresses, because if it were, the material, being flexible, would simply bend or buckle and collapse, which we find does not occur.

It follows from this that a cylindrical tank with a flat bottom uniformly supported is not subject to anything but pure tensile stress.

This simplifies the design very much.

The same result may be obtained from many other considerations, such as those of symmetry.

To calculate the amount of the tension, consider a view of the tank from above (called a *plan* view) at some particular depth where the water pressure is p pounds per square foot, and let D be the diameter of the tank, in feet.

Now, as the pressure of water varies at different depths,

it is best to confine our attention to a strip or section of the tank (see Fig. 1, *b*) of a small height for which the pressure may be considered near enough constant. It is near enough for our purpose to take it 1 ft. high.

Our section of tank is then D feet in diameter, 1 ft. high, and is bounded by a section of tank wall also 1 ft.

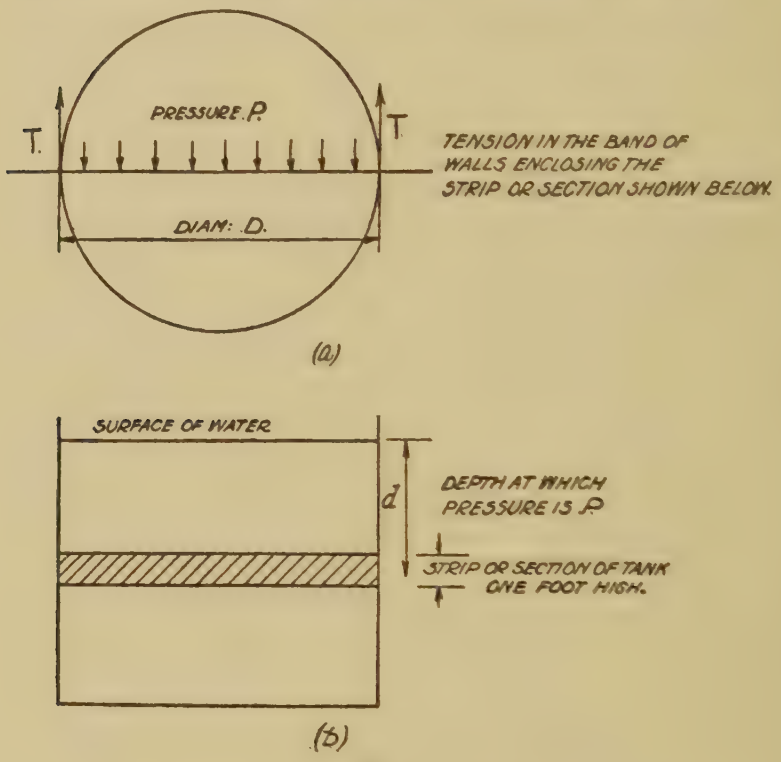


Fig. 1.

high, the tension in which resists the water pressure, pressing against it and tending to burst it.

To find what the actual tension in our section of wall is, let us consider the forces acting across a diameter, as shown on Fig. 1, *a*.

The pressure of water is p lb. per square foot and acts on an area D ft. wide and 1 ft. high, so the total pressure on the diameter of the section is :

$$P = p \times D \times 1 \text{ pounds.}$$

The only forces resisting this are the tensions T in the two sides of the wall of the section. Clearly these forces $2T$ and P must be equal, since if one were the greater, it would pull the tank over, just as in a tug-of-war if one side pulls stronger than the other, movement occurs.

Hence

$$2T = P = pD$$

$$\text{or } T = \frac{pD}{2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

This is a general equation which applies to the tension in any circular receptacle resisting internal pressure, and is used in the design of pipes, boilers, etc.

Now, the pressure of liquid at any depth d is

$$p = wd$$

where w is the weight per cubic foot,
d is the depth in feet,
p is the pressure per square foot.

Substituting this value for p in equation (1)

$$T = \frac{pD}{2} = \frac{wdD}{2} \quad . \quad . \quad . \quad (2)$$

In the case of water, $w = 62\frac{1}{2}$,
so that $T = 31\frac{1}{4}dD \quad . \quad . \quad . \quad (2a)$

In our tank, the maximum pressure is clearly in the lowest foot of the tank and the maximum tension in the lowest foot of wall. In this, the mean depth is $d = 9\frac{1}{2}$ ft. Whence the tension in the lowest foot of wall is

$$\begin{aligned} T &= 31\tfrac{1}{4} \times 9\tfrac{1}{2} \text{ ft.} \times 10 \text{ ft.} \\ &= 2{,}970 \text{ lb.} \end{aligned}$$

When dealing with concrete tanks we must remember that there are two problems :—

(1) To make the tank safe against bursting by the water pressure, and

(2) To make it watertight so that the contents shall not leak out through cracks in the walls.

(1) The first of these requires that there shall be sufficient steel to resist the whole tension without the assistance of the concrete, so that even if the concrete for any reason were to crack, the steel would be sufficient to prevent failure.

Ordinary mild steel to B.S.S. has a breaking strength of about 64,000 lb. per sq. in., but at about half this stress (32,000 lb./sq. in.) it stretches or yields. In ordinary practice, a factor of safety of 2 on the yield point is often used, or a working stress of 16,000 lb. per sq. in. In small tanks, however, it is not desirable to work up to this limit, and a lower stress such as 8,000 lb./in.^{2} is better. The bottom foot of wall has a tension of

$$T = 2{,}970 \text{ lb.}$$

Therefore at a steel stress of 8,000 lb. per sq. in., we shall require an area of steel in this foot of wall of

$$A = \frac{2{,}970}{8{,}000} = \cdot 37 \text{ in.}^2$$

Now $\frac{1}{2}$-in. rods have an area of $\cdot 19$ in.^{2}, so that two $\frac{1}{2}$-in. diameter rods in the foot of wall, that is, $\frac{1}{2}$-in. rods at 6-in. centres, will do nicely.

(2) To make it watertight, it is desirable that the concrete shall not be stressed in tension to such a stress as would crack it.

Good concrete has a breaking stress in tension of about 240 lb. per sq. in. in about three months or about 180 lb. per sq. in. in about one month. A working stress of 60 lb./sq. in. will therefore be safe against cracking.

Now up to the point when the concrete cracks, the concrete and steel will act together, the steel, however, taking fifteen times as much stress as the concrete owing to its higher elastic modulus, just as was explained for Columns (see *Reinforced Concrete Simply Explained*).

Hence if A_c is the area of concrete
and A_s is the area of steel

the *equivalent area of concrete*

$$\text{is } A_E = A_c + 15A_s$$
$$= A_c + A_s + 14A_s$$

Now $A_c + A_s$ is clearly A, the total area of a section (which is made up partly of concrete and partly of steel).

$$\text{Hence } A_E = A + 14A_s.$$

Hence to keep the concrete stress down to 60 lb./sq. in., we can write

$$A_E = \frac{T}{60}.$$

Hence in our example

$$A + 14A_s = \frac{T}{60} = \frac{2,970}{60} = 50 \text{ approx.}$$

The area of steel in one section of wall is, however,

$$A_s = \cdot 38 \text{ sq. in.}$$
$$\text{whence } 14A_s = 5 \cdot 32 \text{ sq. in.,}$$

leaving the area of concrete required

$$A = 50 - 5 \cdot 3 = 44 \cdot 7 \text{ sq. in.}$$

As the depth of our section is 12 in., the necessary thickness required is clearly

$$\frac{44 \cdot 7 \text{ in.}}{12} = 3 \cdot 72 \text{ in.}$$

(3) Apart altogether from tendency to cracking by stress, it is important to observe certain minimum thicknesses.

The object is partly to avoid too great a pressure gradient in the concrete which would entail a risk of gradual oozing over the whole surface of tank (not to be confounded with leakage at cracks), and partly to ensure sufficient thickness to make watertight rebated construction joints (see Fig. 2).

The minimum thicknesses recommended by the author are as follows :—

Depth of Water in Feet.	Thickness in Inches.
3 and under	3
6 ,, ,,	4
9 ,, ,,	5
12 ,, ,,	6
15 ,, ,,	7

and so on.

[In other words

$$t = \frac{d}{36} + 2$$

where t = thickness in inches
d = depth in inches.]

Hence, in the present example, 6-in. thickness meets all requirements.

As the pressure gets proportionately less at a lesser depth from the top, the steel can be reduced, or the bars placed further apart.

Thus a strip 1 ft. high 6 ft. down, instead of 10 ft., would need an area of

$$\cdot 37 \times \frac{6}{10} = \cdot 22 \text{ sq. in.}$$

At this point, therefore, $\frac{3}{8}$-in. rods at 6-in. centres would do.

In the author's view, this should be considered a minimum.

Vertical rods are very desirable. These serve to connect the concrete of different layers, which may be placed on different days. They cannot be calculated, but should be about ·5 per cent. of the concrete, say $\frac{3}{8}$-in. rods at 6-in. centres.

The bottom, if uniformly supported, has no stresses, but $\frac{3}{8}$-in. rods in both directions add little to the cost and serve to resist accidental stresses.

The vertical bars should be bent into the bottom as shown so as to prevent a crack at the internal angle between the sides and bottom.

The concrete should be

$\frac{3}{4}$-in. broken stone or shingle	3 parts
clean sand	$1\frac{1}{2}$,,
Portland cement	1 part

The horizontal joints between successive days' concreting often show leaky places owing to the stones bedding not on to soft mortar, but on to concrete already set.

This danger may be greatly reduced by placing $\frac{1}{2}$ in. of mortar composed of two parts of sand to one of cement on the old work and following the concrete on to this at once.

Another good precaution is to leave rebated finishes, as indicated in the drawing. Fig. 2 gives a complete working drawing.

The horizontal rods are conveniently made in two lengths. These should overlap at each joint by 2 ft. and have a hooked end.

The rods should have 1 in. cover of concrete for protection.

Obviously only a dense watertight stone should be used.

If any leaking should occur, an internal rendering of

1 part cement to $2\frac{1}{2}$ parts sand will probably cure it, but should not be necessary.

The thickness of walls could be reduced to 3 or 4 in.

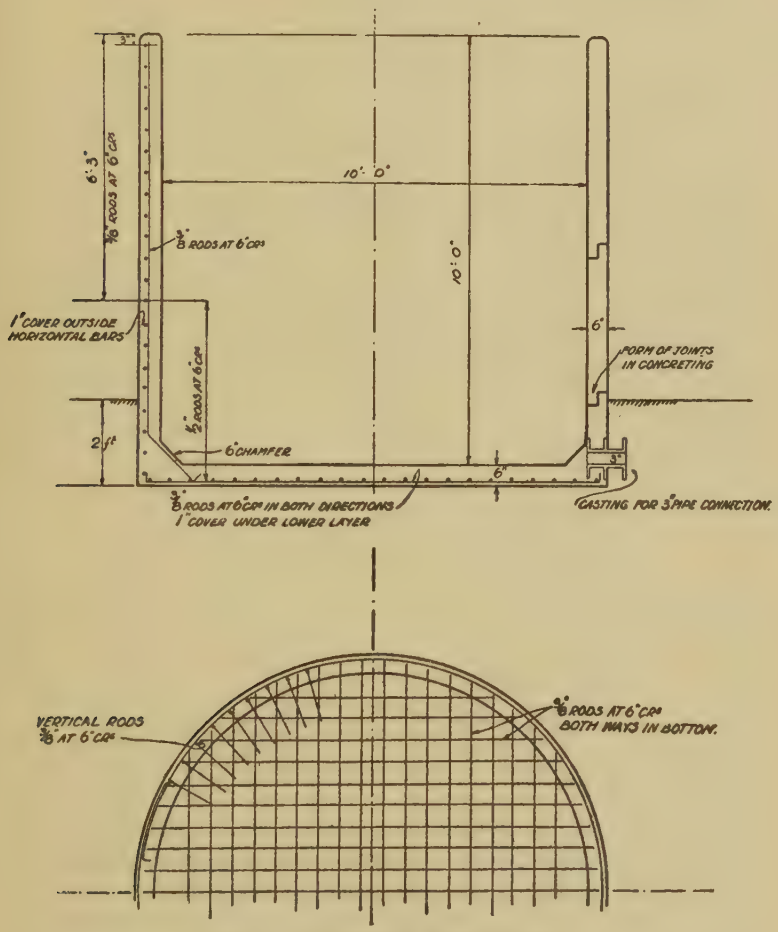


FIG. 2.—A Circular Water Tank.

at the top, but the extra cost of shuttering would probably be as much as that of the concrete saved.

A form of casting for making a good pipe connection is shown.

EXAMPLE II

A SMALL SQUARE TANK

A small tank 10 ft. deep and 10 ft. square.

DESIGN.—The distribution of water pressure against the sides will be exactly the same as explained in the first example, and therefore need not be reiterated.

The walls, in this case, being straight instead of circular will, however, be subject to bending as well as to tension.

Reverting back to Example I, if we make a little model tank of flexible material to a square instead of a round

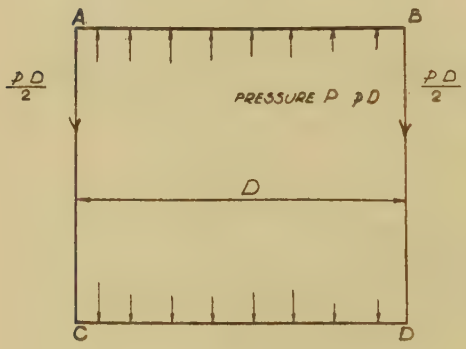


FIG. 3.

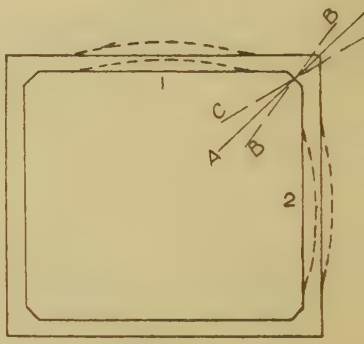


FIG. 4.

design, and fill it with water, it will be found that it will distort at once, and will try to take up a circular form.

Such distortion in the case of a concrete tank would, of course, cause it to fracture, and the sides must be made capable of resisting this bending tendency without appreciable deflection.

Referring to Fig. 3, and considering a horizontal section of wall 1 ft. high at any particular depth, the total pressure on one wall for this strip will clearly be

$$P = pD \text{ lb.}$$

where p = water pressure per square foot at that depth
D = diam. of tank in feet.

This pressure on the side AB is transmitted by the stiffness of the side to the two ends A and B, each end obviously having a reaction of

$$\frac{P}{2} = \frac{pD}{2}$$

which clearly represents a tension of that amount in the walls AC and BD, and balances the pressure in the opposite direction on the side CD.

Similarly pressure on the walls AC and BD produces bending in them and a tension of $\frac{pD}{2}$ in each of the walls AB and CD.

Each wall is therefore subject to a side pressure of pD and a tension of $\frac{pD}{2}$.

Now it was stated in *Reinforced Concrete Simply Explained* that continuous beams may be designed for a bending moment of

$$B = \frac{Wl}{12}$$

where W is the load or total pressure
l is the span, in our case, D.

Substituting, we have

$$B = \frac{pD \times D}{12} = \frac{pD^2}{12}.$$

In the case of our example, the pressure increases uniformly with the depth, as explained in Example I, but the lower portions of the sides receive support from the bottom.

The strip or section with the worst bending therefore occurs some little distance from this bottom, and may be taken at a height of three-tenths of the height of the tank. This in our case is at a height of 3 ft. from the bottom, or at a depth of 7 ft. from the top.

Here the pressure is $7 \times 62\frac{1}{2} = 437$ lb. per sq. ft. and therefore the bending moment is

$$B = \frac{pD^2}{12} = \frac{437 \times 100}{12}$$
$$= 3{,}641\tfrac{2}{3} \text{ ft. lb.}$$
$$= 43{,}700 \text{ in. lb.}$$

Remembering that the resistance moment of a normally reinforced slab is

$$R = 95\ bd^2$$

where b = width of section = 12 in.

d = depth to the steel from the compressed edge,

we may write

$$95\ bd^2 = 43{,}700 \text{ in. lb.}$$

$$d = \sqrt{\frac{43{,}700}{95 \times 12}} = 6{\cdot}2 \text{ in.}$$

A 7-in. slab would therefore be sufficient.

The area of steel required is ·675 per cent. or

$$As = \frac{{\cdot}675}{100} \times 6{\cdot}2 \text{ in.} \times 12 \text{ in.} = {\cdot}5 \text{ sq. in.}$$

This area is required for a strip 12 in. wide, and is that required by considerations of bending only.

Reverting now to the direct tension in the sides of

$$T = \frac{pD}{2}$$

we have $p = 62\frac{1}{2} \times 7$ lb./ft.^{2}

$D = 10$ ft.

whence $T = \dfrac{62\frac{1}{2} \times 7 \times 10}{2} = 2{,}180$ lb.

This tension may be considered as acting uniformly across the slab, half of it as adding to the tension to be carried by the steel, on one side of the slab, the other half as reducing the total compression on the concrete on the other side. This does not pretend to be *strictly* accurate, though it is quite near enough. The accurate treatment of combined bending and tension is given in *Reinforced Concrete Design*, vol. i.

The additional steel required to carry this additional tension is therefore

$$As = \frac{1{,}090}{16{,}000} = {\cdot}07 \text{ sq. in.}$$

Hence the total area of steel required is

$$As = {\cdot}5 + {\cdot}07 = {\cdot}57 \text{ sq. in.}$$

Now three $\frac{1}{2}$-in. rods give this area, and therefore $\frac{1}{2}$-in. rods at 4-in. centres are required.

As regards the thickness of 7 in., it should be noted that had the required thickness come to less than 6 in., as for example in the case of a smaller tank of the same depth, we should still have required to make it at least 6 in. so as to achieve water-tightness, in accordance with the minimum thicknesses from this consideration recommended in Example 1.

A word of explanation is needed, perhaps, in regard to having treated the sides as continuous beams. A little consideration will show that a beam (of which a section of our slab is a special case) is no less continuous because it happens to be bent or cranked at the support.

The distribution of bending moment as between end and support, which is approximately met by designing for $\frac{Wl}{12}$ in both places, corresponds to conditions when the beam is partially fixed in direction at its end.

This was hardly touched on in *Reinforced Concrete Simply Explained*, but is fully gone into in *Reinforced Concrete Design*, vols. i and ii.

In the case of our square tank, the conditions are really better than this, and are, as far as fixity of the corners in direction is concerned, those of beams *fully fixed* in direction at their ends.

This is obvious from symmetry.

Consider one of the corners in Fig. 4. It is clear that the line *AA*, with the tank empty, would remain in exactly the same direction after the tank was filled, because although the deflection of the side 1 would tend to make *AA* turn towards the direction *CC*, the equal deflection of the side 2 would equally tend to make it turn towards the direction *BB*, and the net result would be to leave it where it was.

Now, any book on elementary theory of structures will show that a beam having its ends *perfectly* fixed in direction has a bending moment of $-\frac{Wl}{12}$ at the ends and $+\frac{Wl}{24}$ at the centre.

It may therefore appear that in designing for $+\frac{Wl}{12}$ at the centre, we have designed wastefully.

There is, however, another consideration affecting this, as follows :—

It has been shown (for example, in *Reinforced Concrete Design*, vol. i) that a beam sharply bent is much weaker at the kink or bend than a straight beam of similar dimensions.

Partly for this reason, and also partly because of the high adhesion stresses at the corner, and of the greater difficulty in placing the rods in exactly the right position in the corners, it is unsafe, at any rate in small tanks, to rely entirely on the full fixity at the corners, and therefore the design for $+\frac{Wl}{12}$ at midspan is a prudent precaution.

Coming now to the question of stresses, it will be recalled that the formula

$$R = 95\ bd^2$$

which we used, was based on stresses of 600 lb./in.^{2} in the concrete and 16,000 lb./in.^{2} in the steel, and it will be remembered that in Example 1, a stress limitation of 8,000 lb./in.^{2} in the steel was recommended for small tanks. Some explanation of this apparent anomaly is clearly desirable, and is as follows.

In Example I (a circular tank) the walls were shown to be in pure tension. In this case, the tendency to crack the concrete would extend uniformly through the wall, and any cracks would probably also so extend, leaving a clear passage for leakage.

In Example II, where the walls are subject to bending stresses, the tension exists on one side only, the opposite side being in compression. Hence in this construction, the cracks are not likely to extend beyond the neutral axis, the effect of the compression on the other side being rather to close up any incipient cracks, and hence higher stresses are safe as regards watertightness.

Only near the point of contraflexure (the point separating the negative bending moment at the ends from the positive moment near the centre) is there tension right across the section, and here the stresses are quite small.

This point is generally distant about one-fifth of the span from the ends, and in our case the tension there is

T = 2,180 lb. per ft. wide section and the area of steel.

As = ·57 sq. in. per ft.

whence the stress is

$$t = \frac{2{,}180}{\cdot 57} = 3{,}830 \text{ lb./in.}^2,$$

well within our limiting stress of 8,000 lb./in.² for pure tension.

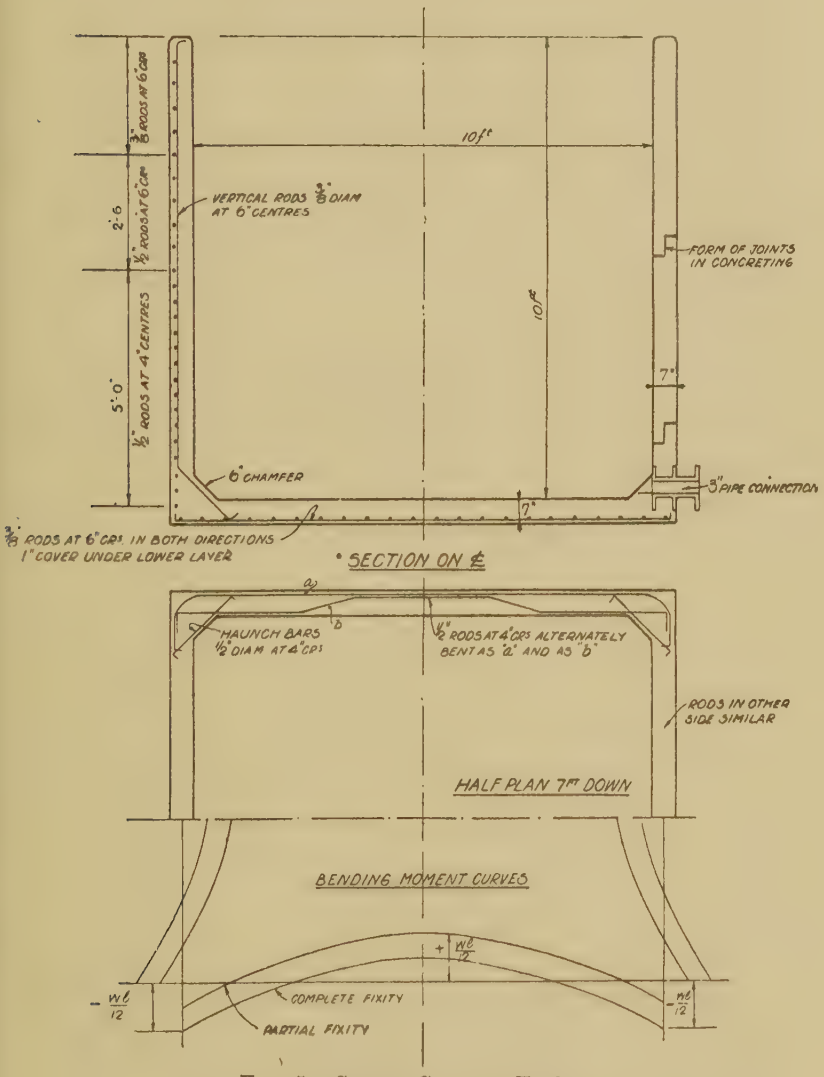

10ft
VERTICAL RODS ⅜" DIAM AT 6" CENTRES
FORM OF JOINTS IN CONCRETING
10ft
7"
6" CHAMFER
3" PIPE CONNECTION
⅜" RODS AT 6" CRS IN BOTH DIRECTIONS 1" COVER UNDER LOWER LAYER
SECTION ON ℄
HAUNCH BARS ½" DIAM AT 4" CRS
½" RODS AT 4" CRS ALTERNATELY BENT AS "a" AND AS "b"
RODS IN OTHER SIDE SIMILAR
HALF PLAN 7ft DOWN
BENDING MOMENT CURVES
COMPLETE FIXITY
PARTIAL FIXITY

FIG. 5.—Square Concrete Tank.

In the design, Fig. 5, it will be noticed that the internal corners are made less sharp by a haunch or chamfer. The object of this is to overcome to some extent the dangers

and disadvantages of sharply bent beams previously referred to.

As regards the bottom, resting directly on the earth, this may be 6 in. thick reinforced with $\frac{3}{8}$-in. rods at 6-in. centres in both directions.

The sides have so far been designed for the worst point, which occurs about 7 ft. down. Clearly the pressure is proportionately less above this point. In our small tank, it is hardly worth while reducing the wall thickness higher up owing to the complication to the centering or timber mould, but the steel may be proportionately spaced out above this point as indicated.

As regards materials, rendering, joints, etc., the remarks in Example I apply.

In Fig. 5, the bending moment curves are given, both for perfect fixity at the ends, and for partial fixity, and the actual curve lies at some point between the two. But in our design we have prudently designed for the worst moment in either curve.

EXAMPLE III

FLOOR SLAB AND BEAMS

Concrete floor slab and beams of single span to cover 100 ft. by 25 ft. (see Fig. 6): live load (for offices) 100 lb. a square foot.

THE desirability of beams follows from the fact that a slab to span 25 ft. direct would be too heavy and expensive.

This may easily be seen as follows: Adopting the rule for slab thicknesses given in *R. C. Simply Explained*,

thickness equal to $\frac{1}{24}$ span,

this would clearly give a slab just over a foot thick, weighing 150 lb. a foot, or 250 lb. a foot with the live load.

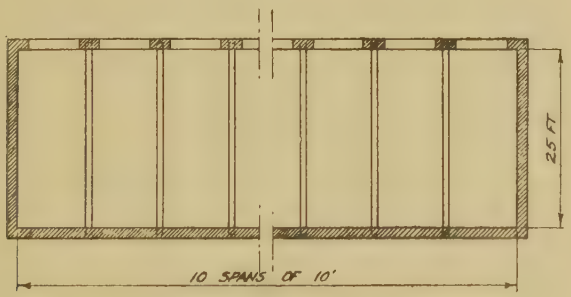


FIG. 6.

Actually this rule only applies to continuous slabs, with a total load of about 175 lb. per foot, so that in the present case a 15-in. slab would be needed.

This would be a very heavy and expensive floor. In general, the arrangement of beams should be so chosen as to give slab thicknesses not exceeding 6 in.

Slab.—We will therefore adopt the arrangement of Fig. 6, namely span the 25 ft. with beams 10 ft. apart,

which will clearly require about a 5-in. slab, working from our rule already referred to.

A 5-in. slab should have $\frac{7}{16}$-in. rods at 6-in. centres (see *R.C.S.E.*, p. 24).*

As a check on this simple design, let us calculate the slab out more fully.

The loads for a 5-in. slab will be

live	load	100	lb. per sq. ft.
dead	,,	$62\frac{1}{2}$	,, ,, ,,
total	,,	$162\frac{1}{2}$	,, ,, ,,

The bending moment may be taken at $\frac{Wl}{12}$ for continuous beams or slabs (see Fig. 6 (*e*) of *R.C.S.E.*).†

So that $B = \frac{Wl}{12}$ on a strip 1 ft. wide and 10 ft. span ($b = 12$ in.)

$$= \frac{(162\frac{1}{2} \times 10 \text{ ft.}) \times 120 \text{ in.}}{12}$$

$$= 16{,}250 \text{ in. lb.}$$

Now with ·675 per cent. of steel and usual stresses, the resistance moment is

$$R = 95\ bd^2 \text{ in. lb.}$$

so that $$95\ bd^2 = 16{,}250$$

and $$d = \sqrt{\frac{16{,}250}{95 \times 12}} \text{ (since } b. = 12 \text{ in.)}$$

$$= 3{\cdot}8 \text{ in.}$$

Area of steel $= {\cdot}675$ per cent. of effective concrete

$$= \frac{{\cdot}675}{100} \times 3{\cdot}8 \text{ in.} \times 12 \text{ in.}$$

$$= {\cdot}31 \text{ sq. in. per ft. width.}$$

Now the area of a $\frac{7}{16}$-in. rod is ·15 sq. in., so that $\frac{7}{16}$-in. rods at 6-in. centres gives ·3 sq. in. per ft.

* NOTE.—In these examples, *R.C.S.E.* refers to *Reinforced Concrete Simply Explained* (Oxford University Press).

† The reader is reminded that we are confining ourselves deliberately to simple solutions of sufficient accuracy for work where the utmost economy is not required, this being only securable by an expert designer.

To the effective depth of 3·8 in. has of course to be added enough concrete to cover it. So that a 5-in. slab with $\frac{7}{16}$-in. rods at 6-in. centres will do well.

As a matter of fact, the slab could be as little as

$$3{\cdot}8 + {\cdot}7 = 4{\cdot}5 \text{ in.}$$

in thickness, ·7 being the cover (·5 in.) plus half the diameter of the rod.

Now this calculation applies strictly speaking to the eight interior slab panels only. The two end panels are subject to a greater moment, because these are continuous at one support only.

In general, these should be designed for

$$B = \frac{Wl}{10},$$

an increase of 20 per cent.

If the slab is redesigned for this increased moment an increased thickness and steel area will result.

It is often undesirable to vary the slab thickness in the end panels, and in such cases it is satisfactory to obtain the whole increased strength by increasing the steel by 40 per cent.

In our present case, this would be met by putting the rods $4\frac{1}{3}$-in. centres instead of 6-in. centres, though for practical reasons 4-in. centres would probably be adopted.

The arrangement of slab bars is shown in Fig. 7, and it will be seen that the negative moments over the supports $\left(\text{equal to } \frac{Wl}{12}\right)$ are fully provided against.

Beams.—Load per beam

	lb.
25 ft. × 10 × $162\frac{1}{2}$	= 40,625
wt. of beam	
say 25 ft. × 300 lb.	= 7,500
	48,125

It will be seen that it is necessary to assume a reasonable weight for the beam, though this requires correction after the calculations are further advanced.

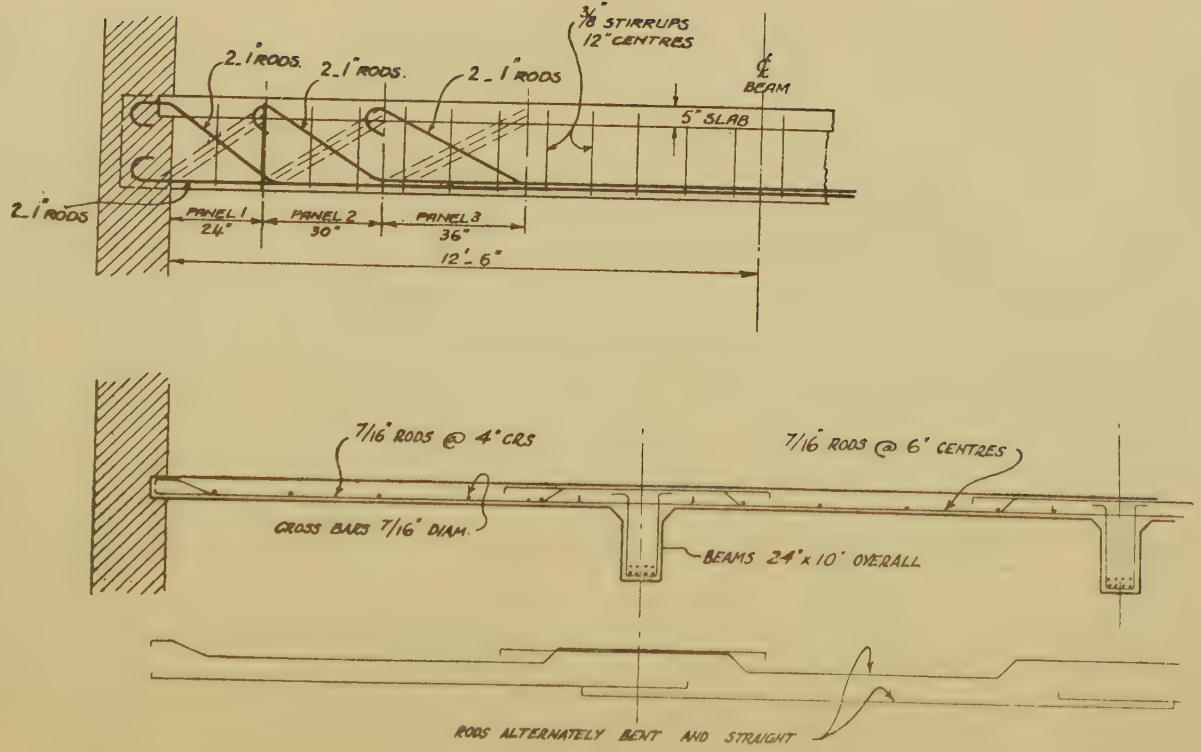

3/8" STIRRUPS
12" CENTRES
2-1" RODS.
2-1" RODS.
2-1" RODS
℄ BEAM
5" SLAB
2-1" RODS
PANEL 1
24"
PANEL 2
30"
PANEL 3
36"
12'-6"
7/16" RODS @ 4" CRS
7/16" RODS @ 6" CENTRES
CROSS BARS 7/16" DIAM.
BEAMS 24" x 10" OVERALL
RODS ALTERNATELY BENT AND STRAIGHT

FIG. 7.

Bending moment for simply supported beams uniformly loaded (see *R.C.S.E.*, Fig. 6 (*d*)),

$$\text{is } B = \frac{Wl}{8}$$

$$= \frac{48{,}125 \times 300 \text{ in.}}{8}$$

$$= 1{,}805{,}000 \text{ inch lb.}$$

Referring to the author's table of Standard T-beams (see *R.C.S.E.*, p. 35), it will be seen that a beam 24 in. × 10 in. overall with eight 1-in. rods gives an ample resistance moment.

Now we must see whether our concrete slab is sufficiently strong to act as the compression member to our beam.

Referring back to our table of Standard Beams (*R.C.S.E.*, p. 35) it will be seen that this beam needs a slab 7·9 in. thick and 42·3 in. broad.

Actually our slab is 5 in. thick and, according to the L.C.C. requirements, the safe width may be taken as the least of the following three :—

(*a*) distance between beams	= 120 in.
(*b*) one-third the span	= 100 in.
(*c*) twelve times slab thickness	= 60 in.

In our case, therefore, we supply a slab 5 in. × 60 in., which is practically equal to the 7·9 in. × 42·3 in. asked for by the table.

In any case, the reasoning on which requirement (*c*) above is based is very obscure, and the author would not mind taking a greater width of slab as acting.

Coming now to the Shearing Resistance, the shear per beam at the end is

$$S = 24{,}063 \text{ lb.}$$

If we adopt the arrangement of bent-up rods shown in Fig. 7, increasing the slope towards the end where the shear is greatest, the shear resistance will be

Shear Resistance in Panel 1 due to

	lb.
Inclined tension $= 2 \times \cdot 78 \times 10{,}000^* \times \frac{20 \text{ in.}}{30 \text{ in.}}$	$= 10{,}400$
Inclined compression	$= 10{,}400$
Stirrups $\cdot 22 \times 10{,}000^* \times \frac{20}{12}$ . . .	$= 3{,}600$
	24,400 lb.

Shear Resistance in Panel 2 due to

Inclined tension $2 \times \cdot 78 \times 10{,}000^* \times \frac{20}{36}$.	$= 8{,}600$
Inclined compression	$= 8{,}600$
Stirrups	$= 3{,}600$
	20,800 lb.

$$\text{Actual Shear} = 24{,}063 \times \frac{10\frac{1}{2}}{12\frac{1}{2}} = 20{,}200 \text{ lb.}$$

Shear Resistance in Panel 3 due to

Inclined tension $2 \times \cdot 78 \times 10{,}000 \times \frac{20}{42}$.	$= 7{,}400$
Inclined compression	$= 7{,}400$
Stirrups	$= 3{,}600$
	18,400 lb.

$$\text{Actual Shear} = 24{,}063 \times \frac{8}{12\frac{1}{2}} = 15{,}500 \text{ lb.}$$

Shear Resistance in Panel 4

Inclined tension in concrete 20 in. × 10 in. × 60 in. $= 12{,}000$ lb.

$$\text{Actual Shear} = 24{,}063 \times \frac{5}{12\frac{1}{2}} = 9{,}700 \text{ lb.}$$

The treatment of Shear follows that fully explained in *R.C.S.E.*, Chap. IV.

* We are adopting a low stress in the shear members, as recommended in *R.C.S.E.* This is because high stresses near bends involve excessive compressive stresses in the concrete.

EXAMPLE IV

WAREHOUSE FLOOR ON CONCRETE COLUMNS

Warehouse floor 100 ft. × 100 ft. carried on concrete columns. Load 300 lb. a foot inclusive. Two stories of 12 ft. with a flat roof over.

The first decision that needs to be made is the centres of columns.

Considerations of economy in this matter generally fix the most economical spacing as, roughly, the distance between floors.

As in the present case, however, this would be so close as to be a serious obstruction, and the best solution must be a compromise between economy and utility, which will need deciding on the merits of each case specially.

In the present case we will settle on 25-ft. spacings in both directions, which gives fine open spaces.

The next decision is the arrangement of beams. Probably a flat slab or mushroom design would need to be considered, but this can hardly be considered a " simple example," and must be left to more experienced hands at present. In any case, it by no means always furnishes the best solution.

Remembering that as a general rule a good design should not give a slab thickness greater than about 6 in., it is clear that the slab cannot span the 25 ft. between columns.

Even with single intermediate beams, the slab would be about 12 ft. 6 in. span and therefore just over 6 in. thick for office loads, and clearly more for the present warehouse loads.

We shall therefore be well advised to have two secondary beams between those on the lines of columns, thus dividing every square panel up into three.

The arrangement is now as indicated on Fig. 8.

In this example, we will not complicate the work by introducing staircases, lifts, etc.

We will also assume the flat roof is to be designed for

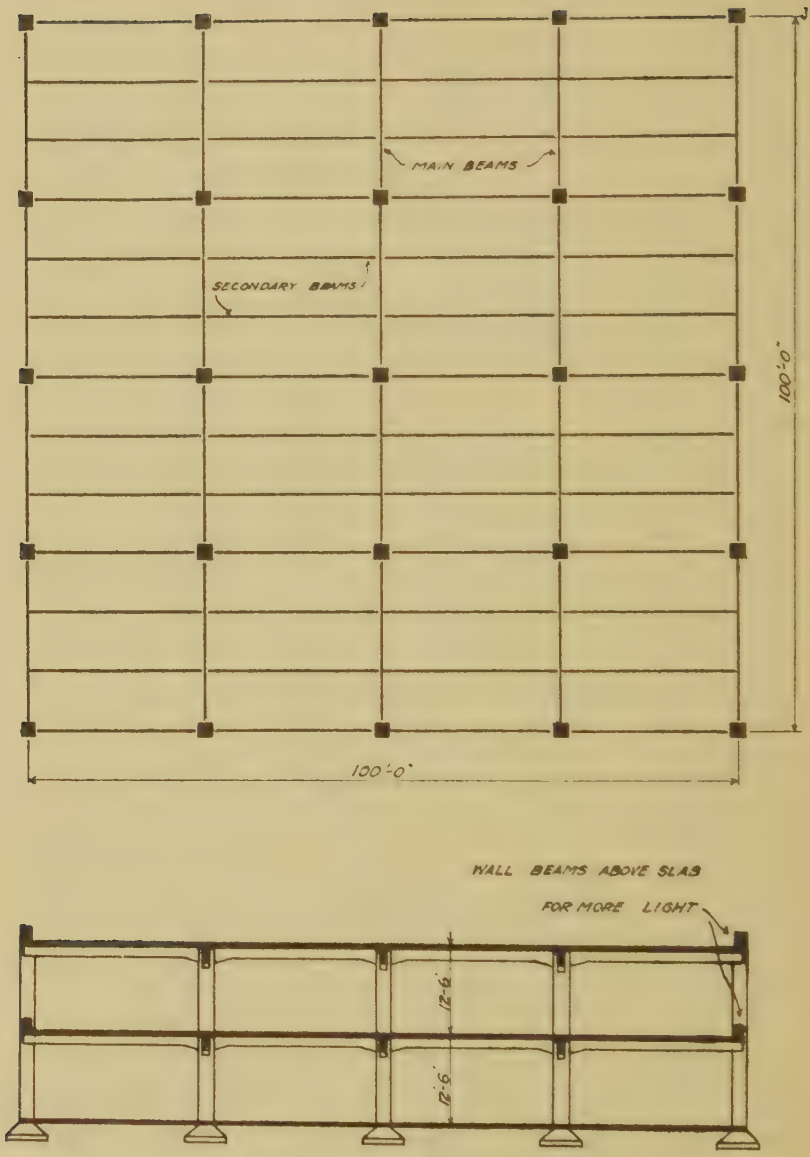


Fig. 8.—General Plan and Section of Floors.

the heavy floor load, so that it could at a later date be used as a floor and have a light roof built on it.

We can now proceed as follows :—

Floor Slab.— $\text{Span} = \frac{25}{3}$ ft. $= 8$ ft. 4 in.

Bending moment : $B = \frac{Wl}{12} = \frac{(300 \times 8\frac{1}{3}) \times 100 \text{ in.}}{12}$

$= 20{,}800$ in. lb. per ft. width.

With stresses of 16,000 and 600 lb./in.^{2} in steel and concrete respectively and ·675 per cent. of steel, we have the resistance moment : $R = 95\ bd^2$; whence, remembering $b = 12$ in., and $R = B$,

$$d = \sqrt{\frac{20{,}800}{95 \times 12}} = 4\tfrac{1}{4} \text{ in.}$$

Hence a 5-in. slab gives reasonable cover. Area of steel per ft. width of slab

$$A = \frac{\cdot 675}{100} \times 4\tfrac{1}{4} \times 12$$

$$= \cdot 345 \text{ in.}^2$$

$\frac{1}{2}$-in. rods at 7-in. centres gives this area.

The slab bars should be alternately straight along the bottom, and bent up over the support, as explained on p. 23 of *R.C.S.E.*

The cross bars may be $\frac{1}{2}$-in. rods 28 in. apart (i.e. four times the spacing of the main bars).

Secondary Beams.—Span $= 25$ ft.

Load per beam $= 25$ ft. $\times$ 8 ft. 4 in. $\times$ 300 lb./ft.^{2}

$= 62{,}500$ lb.

Bending moment : $B = \frac{Wl^*}{12} = \frac{62{,}500 \times 300 \text{ in.}}{12}$

$= 1{,}562{,}500$ in. lb.

Referring to the table of standard T-beams given in *R.C.S.E.*, p. 35, it appears that something between a 22×8 and a 24×10 is required.

We will, therefore, adopt the larger of these which is on the side of safety. This has eight 1-in. rods in two layers.

We notice from *R.C.S.E.*, p. 35, that the compression slab needs to be 7·9 in. thick and 42·3 in. wide—actually

* See footnote on p. 32.

it is 5 in. thick and 100 in. wide, which gives a more ample area.

So far we have only considered moments at midspan. Coming now to the negative moment at the support, this will have about the same value.

$$B = \frac{Wl^*}{12} = 1{,}562{,}500 \text{ in. lb.}$$

Remembering that most of the secondary beams are supported, not by columns, but by main beams, it will

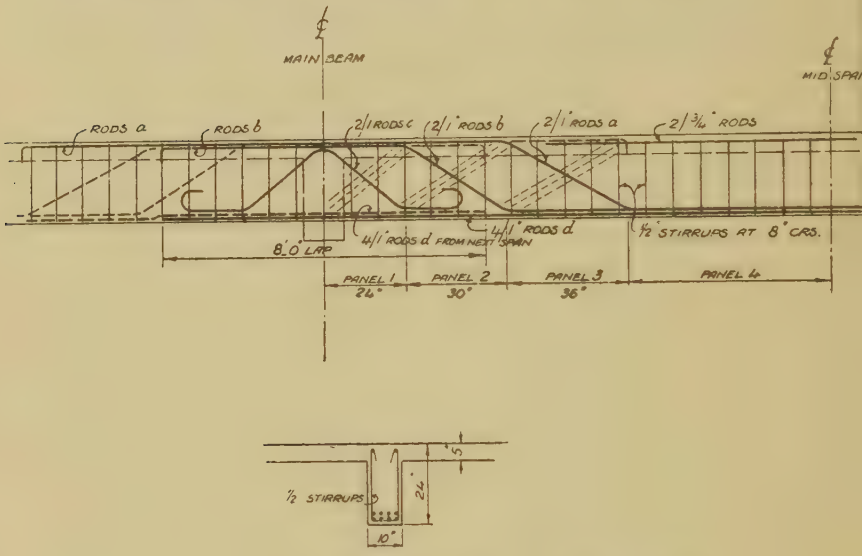


FIG. 9.—Details of Reinforcement of Secondary Beam.

clearly be difficult to provide an appreciable haunch at the end.

We notice that $\frac{B}{bd^2}$ gives the figure

$$\frac{1{,}562{,}000}{10 \times 22^2} = 322$$

instead of the usual 95 bd^2 for rectangular beams.

If we have eight 1-in. rods on the tension side (on top at the support) this will clearly be ample, since this was

* In these examples, only approximate values are used. For accurate values, see *Reinforced Concrete Design*, vol. ii.

ample for midspan, and both the moment and depth at the end are unchanged.

The compression side would then be greatly overstressed, but we can relieve it by using rods on the compression side (the bottom).

As regards the number and size required, the L.C.C. allow an area equal to that on the tension side. For large percentages, this has little justification, but we will accept it for the purpose of these simple examples.

Coming now to Shear, we will consider the beam divided into panels, with the rods arranged as in Fig. 9.

PANEL 1.—*Shear* $= 12\frac{1}{2} \times (8\frac{1}{3} \times 300) = 31{,}250$ lb.

Shear Resistance due to

	lb.
Inclined tension members (two 1-in. rods) $1{\cdot}56 \times 10{,}000 \times \frac{20}{30}$	$= 10{,}400$
Inclined compression	$= 10{,}400$
Stirrups ($\frac{1}{2}$-in. loops at 8-in. centres) $\cdot 4 \times 10{,}000 \times \frac{20}{8}$	$= 10{,}000$
	30,800 lb.

This agrees well enough with the actual Shear.

PANEL 2.—*Shear* $= 31{,}250 \times \frac{10\frac{1}{2}}{12\frac{1}{2}} = 26{,}500$ lb.

Shear Resistance due to

	lb.
Inclined tension . $1{\cdot}56 \times 10{,}000 \times \frac{20}{36}$	$= 8{,}600$
Inclined compression	$= 8{,}600$
Stirrups	$= 10{,}000$
	27,200 lb.

PANEL 3.—*Shear* $= 31{,}250 \times \frac{8}{12\frac{1}{2}} = 20{,}000$ lb.

Shear Resistance due to

	lb.
Inclined tension . $1{\cdot}56 \times 10{,}000 \times \frac{20}{42}$	$= 7{,}400$
Inclined compression	7,400
Stirrups	10,000
	24,800 lb.

PANEL 4. *Shear* = $31{,}250 \times \frac{5}{12\frac{1}{2}} = 12{,}500$ lb.

Inclined tension in concrete
20 in. × 10 in. × 60 = 12,000 lb.
Resistance of Stirrups . . . . 10,000 ,,

Hence either of these practically meets the requirements alone. They must not be added.

The treatment of Shear as given above follows exactly that explained fully in *R.C.S.E.*

The stress in stirrups and shear members has been confined to 10,000 because it is impossible to develop high stresses in the neighbourhood of ends of rods or of bends without producing excessive stresses in the concrete.

Note carefully the lap of 4 ft. on either side of the centre line given to the bars on the compression side. This is necessary if they are to be capable of resisting the 16,000 lb. in.* to which the stress in them may approximate.

The same applies also to the bent bars carried beyond the support, except that here some of the bars are carried a little further to satisfy the requirements of the negative moments.

Interior Main Beams.—These carry concentrated loads at the third points, each load being the total load from a secondary beam (62,500 lb.).

If the beam were freely supported, the bending moment would be $B = \frac{Wl}{6}$ at the centre and nothing at the ends.

With continuous beams, it is desirable to make both ends and centre strong enough to resist two-thirds of this moment, i.e.

$$B = \frac{Wl}{9}$$

that at the centre being of course positive and that at the ends negative.*

This gives

$$B = \frac{Wl}{9} = \frac{125{,}000 \text{ lb.} \times 300 \text{ in.}}{9} = 4{,}167{,}000 \text{ in. lb.}$$

Referring to the author's Standard **T** beams (*R.C.S.E.*, p. 35), this clearly requires a beam 30 in. × 12 in. overall with ten 1¼-in. bars.

* This is a rough approximation only. For exact values, see *Reinforced Concrete Design*, vol. ii.

As regards the strength of the compression boom (formed by the floor slab), the table indicates 66 in. wide and 9·9 in. deep as being required.

Our slab is only 5 in. thick, and the available width is limited to (see *R.C.S.E.*, p. 34)

(*a*) distance between beams = 300 in.
(*b*) one-quarter the span = 75 in.
(*c*) twelve times slab thickness = 60 in.

whichever is less.

Actually, then, our slab is approximately the correct width, but only half the required depth.

This means it will carry about three-quarters the necessary compression, as an inspection of the stress diagram indicates that the upper half of the compression triangle has three-quarters the total area.

Compression steel must therefore be used to carry about one-quarter the total compression.

The total compression is

$$C = \frac{B}{a} = \frac{4{,}167{,}000}{24{\cdot}2} = 172{,}000 \text{ lb.}$$

The concrete slab (with an average stress of 450/lb. in.^{2}) carries

$C = 60$ in. $\times$ 5 in. $\times$ 450 = *135,000*
leaving the steel to take *37,000* lb.

The additional stress on each square inch of compression steel will be

$$c = 14 \times 450 = 6{,}300 \text{ lb./in.}^2$$

if it is centrally disposed in the slab, so that the area required is

$$A = \frac{37{,}000}{6{,}300} = 5{\cdot}9 \text{ in.}^2$$

Five $1\frac{1}{4}$-in. rods give

$$5 \times 1{\cdot}22 = 6{\cdot}1 \text{ in.}^2$$

We must remember to insert in the slab, bars 60 in. long crossing the main beam, to tie the slab to it for the width we are using; $\frac{1}{2}$-in. rods at 7-in. centres will be suitable (as used for the main slab reinforcement). (See *R.C.S.E.*, par. 48, p. 34.)

Negative Moment at Support.—As already stated, we will take

$$B = -\frac{Wl}{9} = -4{,}167{,}000 \text{ in. lb.}$$

Adopting the rule given in *R.C.S.E.* (p. 37) for the author's standard **T** beams we use a haunch to give an overall depth at the column of

$$N = 2 \times 30 = 60 \text{ in.}$$

and let this die away at a distance of $\frac{l}{8} = 3$ ft. $1\frac{1}{2}$ in. from the centre line.

As a check on this, note this would give a resisting moment of

$$R = 95\,bd^2 = 95 \times 12 \times 57^2$$
$$= 3{,}700{,}000 \text{ in. lb.}$$

This is just under what is required, but the presence of more steel at the top than ·675 per cent. will make up the difference.

Actually provided ten $1\frac{1}{4}$-in. rods $= 12{\cdot}2$ in.^{2}

·675 per cent. would be $\frac{{\cdot}675}{100} \times 12 \times 57 = 4{\cdot}6$,,

Shear.—The Shear, with this load distribution, is uniform from support to the nearest third point, and has a value equal to half the load on the beam, namely,

$$S = 62{,}500 \text{ lb.}$$

If the bars are bent as in Fig. 10, then the shear resisted by the bent-up bars is

$2 \times 1{\cdot}22 \times 10{,}000 \times \frac{24 \text{ in.}}{30 \text{ in.}}$ . .	= 19,600 lb.
Inclined compression . . .	= 19,600 ,,
$\frac{3}{4}$-in. stirrups at 12-in. centres	
$2 \times {\cdot}44 \times 10{,}000 \times \frac{24 \text{ in.}}{12 \text{ in.}}$ . .	= 17,600 ,,
	56,800 lb.

Having regard to the conservative basis of design, this would be satisfactory.

The centre third has properly no shear when the load is uniform. But unequal loading can produce some, and a minimum reinforcement of $\frac{1}{2}$-in. stirrups at 12-in. centres is desirable.

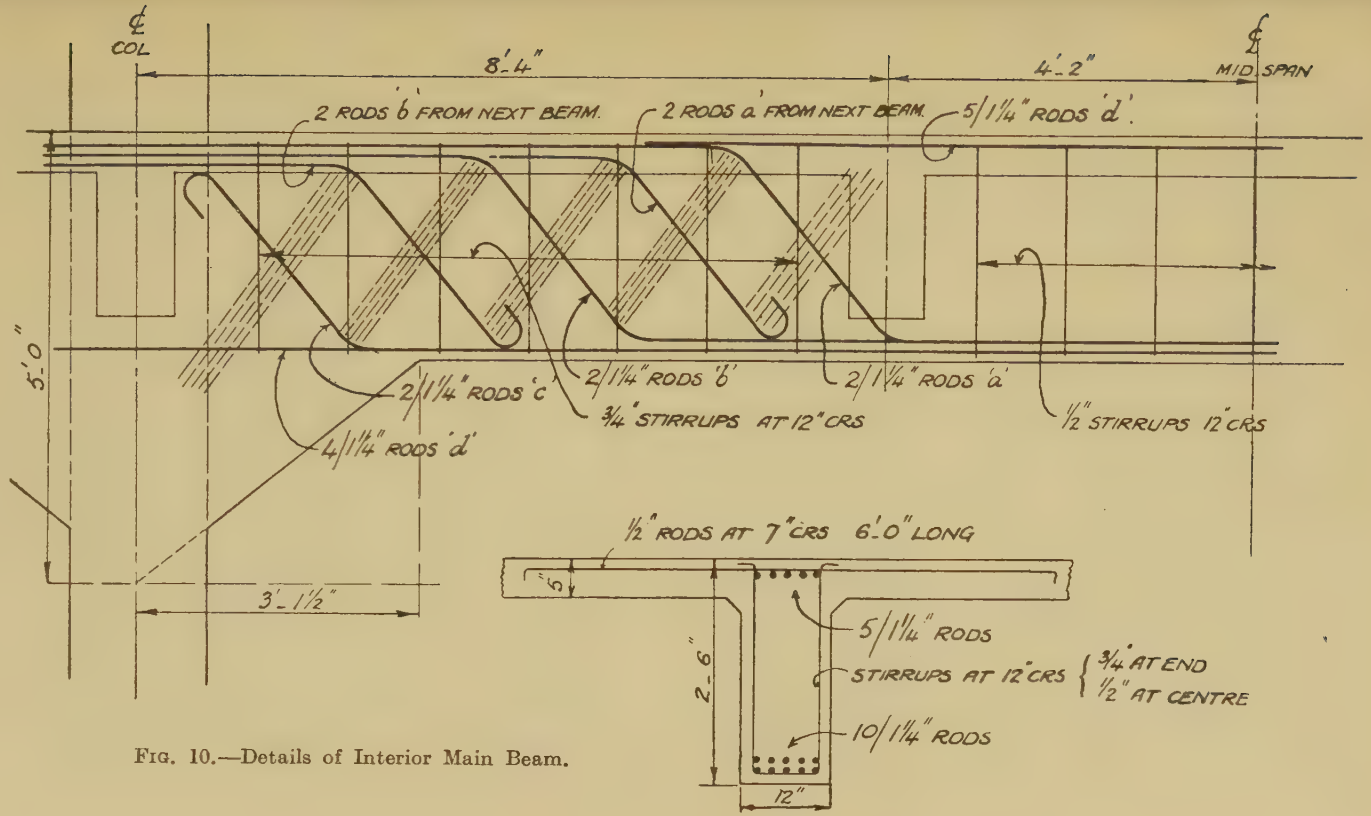


FIG. 10.—Details of Interior Main Beam.

Main Beams in Walls.—These differ from the others in that they carry only about half the floor load, but also carry a wall.

Taking two-thirds the free moment, both at centre and support, the moment would be

$$\text{from floor } \frac{Wl}{9} = \frac{62{,}500 \text{ lb.} \times 300 \text{ in.}}{9} = 2{,}080{,}000 \text{ in. lb.}$$

$$\text{from wall } \frac{Wl}{12} = \frac{[25 \text{ ft.} \times 450] \times 300 \text{ in.}}{12} = 280{,}000 \text{ in. lb.}$$

2,360,000 in. lb.

This assumes the wall to consist of the concrete beam 3 ft. high, 12 in. thick, the weight of window being negligible, and giving a maximum of light.

Our beam is then 3 ft. 5 in. overall.

Treating this as a rectangular beam singly reinforced (because clearly at midspan the slab will not be available on the compression side), we have the resisting moment

$$R = 95bd^2 = 95 \times 12 \times 38^2$$
$$= 1{,}650{,}000 \text{ in. lb.}$$

This being insufficient, the beam will need reinforcing on the compression side. We will therefore design it on the L.C.C. rule of using equal area of steel top and bottom, in which case the area of steel required is

$$A_s = \frac{2{,}360{,}000}{\cdot 88 \times 38 \times 16{,}000} = 4{\cdot}45 \text{ in.}^2$$

Six 1-in. rods gives $6 \times \cdot 78 = 4{\cdot}68$ in.^{2}

Reverse Moment of Support.—The moment being the same, and the beam being symmetrically reinforced, the centre section will apply equally well at the ends.

Here we also have a slab to help in compression, but it is all one side and therefore needs special consideration owing to its eccentricity.

Shear.—The total Shear is

From secondary beam	31,250 lb.
From wall ($12\frac{1}{2} \times 450$)	5,600 ,,
	36,850 lb.

Adopting the arrangement of Fig. 11 and designing as before, the shear resistance due to

Inclined rods $= 2 \times \cdot 78 \times 10{,}000 \times \frac{36 \text{ in.}}{58} = 9{,}700$ lb.

Inclined compression . . . . = 9,700 ,,

Stirrups $\frac{5}{8}$ in. at 12-in. centres

$2 \times \cdot 3 \times 10{,}000 \times \frac{36}{12}$ . . . = 18,000 ,,

37,400 lb.

A special point arises in this beam which is usually absent.

The load being applied at the bottom instead of at the top clearly increases the tension in the shear reinforcement, and care has to be taken in a beam of this kind to see that the bottom loads are well slung up.

In the case of the point loads from the secondary beams, this is best done by adding special hangers as shown, thus carrying the load up to its normal position at the top of the beam.

As the load from each secondary beam is 31,250 lb., four sections of 1-in. rod would give a stress of

$$\frac{31{,}250}{4 \times \cdot 78} = 10{,}000 \text{ lb./in.}^2$$

This is a safe and desirable stress in such a member.

Having now designed the secondary and main beams, we can turn to the columns and footings. It should, however, be noted that all the beams already calculated were beams continuous at both ends with beams in adjacent spans. Actually, however, only half the beams are continuous at both ends, the remainder being continuous at one end, but resting on a wall beam or a wall column at the other end.

The effect of continuity at both ends was to reduce the moment for which the beams were to be designed from $\frac{Wl}{8}$ to $\frac{Wl}{12}$. When continuous one end only, instead of two, only half this reduction should be made, namely, from $\frac{Wl}{8}$ to $\frac{Wl}{10}$. This will be done with sufficient accuracy by increasing the area of steel in all the end beams by multiplying the area in the interior beams by the fraction

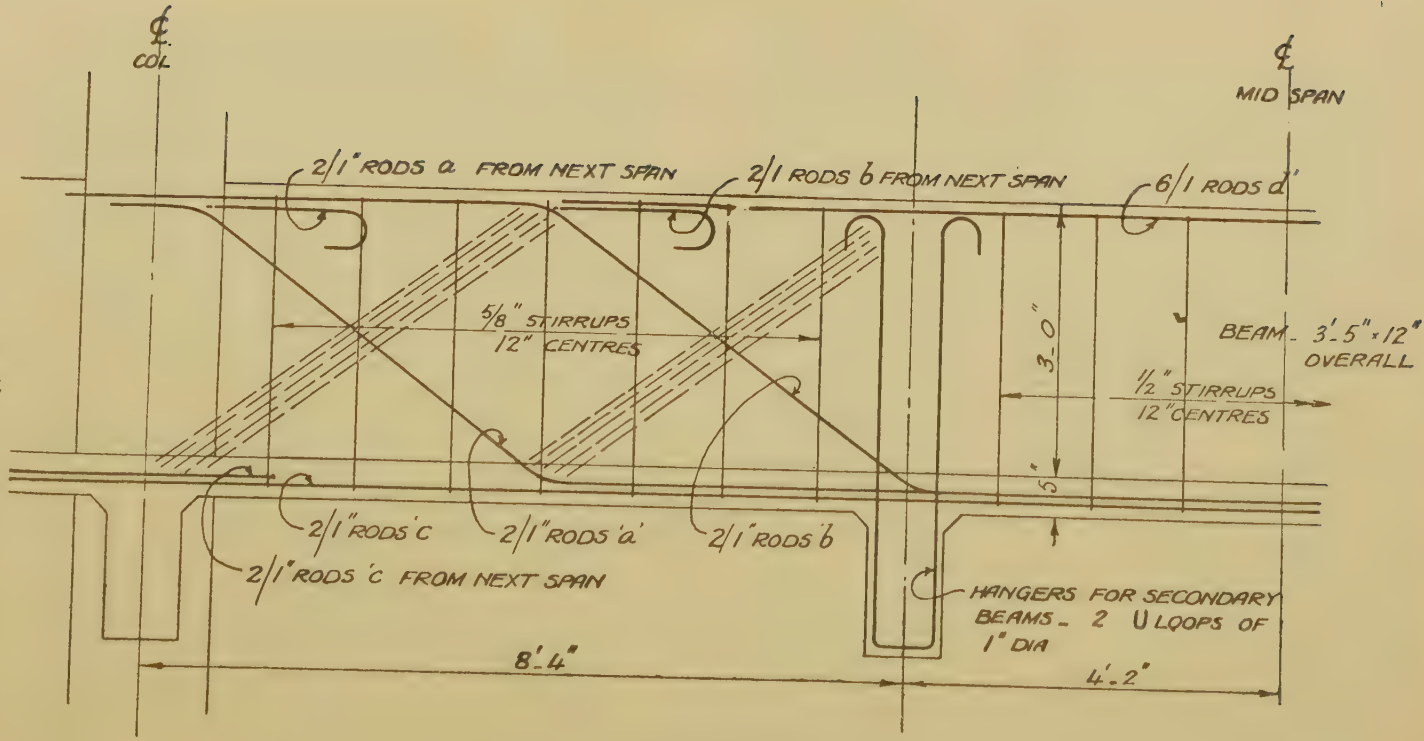


FIG. 11.—Details of Main Wall Beams.

$\frac{12}{10}$, leaving the arrangements of bars the same. The shear is not substantially changed,* so this will meet all the requirements of the case.

Columns.†—There are really four types of columns, nine centre ones (*a*), six wall columns carrying one main beam and two secondary beams (*b*), six wall columns carrying two main beams and one secondary beam (*c*), and four corner columns (*d*).

Column Type (*a*)

This column clearly carries at each floor

2 main beam reactions of 62,500 lb.	= 125,000 lb.
2 secondary beam reactions of 31,250 lb.	= 62,500 ,,
	187,500 lb.

As a check on this, the total load on each panel is clearly

$$25 \text{ ft.} \times 25 \text{ ft.} \times 300 \text{ lb./ft.}^2 = 187{,}500 \text{ lb.}$$

If there were no bending in the columns, or unequal loading of the floors, the columns would be designed for a concentric load of 187,500 lb. for the upper length and 375,000 lb. for the lower.

Stressing the columns to 600 lb./in.^{2}, this would require equivalent areas of

$$\frac{187{,}500}{600} = 312 \text{ sq. in. for the upper length}$$

and $$\frac{375{,}000}{600} = 624 \text{ sq. in. for the lower length.}$$

Referring to *R.C.S.E.*, p. 53, it will be seen these areas are given approximately by standard 16-in. and 24-in. square columns respectively, having four 1-in. rods and eight 1-in. rods respectively.

It will, however, be interesting to see what stresses may be induced by bending of the beams when the panels on the two sides of any column are unequally loaded, the worst bending occurring when the main beam on one side

* These approximations are essential to a "simple example." More accurate moment and shear determinations are given in *Reinforced Concrete Design*, vols. i. and ii.

† Before studying this, the reader is recommended to re-read *R.C.S.E.*, Chap. V., and also the Appendix to the present volume.

is fully loaded and that on the other is subject to the dead load only.

Let us consider the upper length first and follow the treatment given in *R.C.S.E.*, Chap. V. The load on the column, under these circumstances, will, of course, be less, and will be as follows :—

	lb.
from the loaded main beam, as before . .	62,500
from the unloaded main beam, dead load only	20,833
from the two secondary beams, dead load .	20,833
half live load	20,833
say,	125,000

In this calculation it is assumed that of the total load of 300 lb. per sq. ft., one-third is dead load (i.e. weight of slab and beams, etc.) and the remainder live load. This gives a dead load of 100 lb. per sq. ft., which can be checked with the weight of the slab and beams already calculated, and found to be practically correct.

The two secondary beams will carry only half the usual live load, as they will only be loaded on one side (i.e. towards the loaded main beam) and unloaded on the other.

The actual direct stress due to the load is therefore only

$$\frac{125{,}000}{300} = 416 \text{ lb./in.}^2,$$

300 being the equivalent area of the standard 16 × 16 column (see *R.C.S.E.*, p. 53).

Coming now to the stress due to bending, the moment of inertia of the main beam (standard 30 in. × 12 in. T beam) is 78,000 in.^{4} (see *R.C.S.E.*, p. 35) and its length (or span) 300 in., while the moment of inertia of the standard 16-in. square column (*R.C.S.E.*, p. 53) is 7,090 in.^{4} and its length 150 in.

F_p, the fixity factor for an interior column, may be taken as 4 (*R.C.S.E.*, p. 55),

$$\text{whence } \frac{F_p \cdot S_p}{S_b} = \frac{4 \times \frac{7{,}090}{150}}{\frac{78{,}000}{300}}$$

$$= \cdot 73$$

Referring to Table III (*R.C.S.E.*, p. 55), it will be seen that the corresponding moment in the pillar is about midway between

$$\frac{Wl}{117} \text{ and } \frac{Wl}{60}, \text{ say } \frac{Wl}{88}$$

where W is the *live* load on the beam and l the span.

So that $B = \frac{Wl}{88} = \frac{83{,}333 \times 300 \text{ in.}}{88} = 284{,}000$ in. lb.

This needs to be multiplied by $\frac{12}{9}$ owing to third point instead of uniform loading for which Table III, *R.C.S.E.*, p. 55, was prepared, giving

$$B = \frac{12}{9} \times 284{,}000 = 376{,}000 \text{ in. lb.}$$

$$C = \frac{Bd}{2I} \text{ (see } R.C.S.E.\text{)}$$

$$= \frac{376{,}000 \times 16 \text{ in.}}{2 \times 7{,}090} = 426 \text{ lb./in.}^2$$

This can be reduced for depth of beam and haunch to $c = 426 \times \frac{4 \text{ ft. } 0 \text{ in.}}{6 \text{ ft. } 3 \text{ in.}} = 272$ (see Fig. 12, and explanation given in Appendix).

Hence, the maximum stress is

416 direct + 272 bending = 688 lb./in.^{2} total.

The bending in the lower length of column may occur with the upper floor fully loaded, and the lower floor loaded on one side only, as before, in which case the load on the column will be

	lb.
load from upper floor (as before) . .	187,500
,, ,, lower floor (as before) . .	125,000
Total	312,500

Direct stress $= \frac{312{,}500}{664} = 470$ lb./in.^{2}, 664 being the equivalent area of the 24-in. square standard column (*R.C.S.E.*, p. 53).

In this case the bending of the beam in first floor causes

bending in, and is therefore resisted by both the upper and lower lengths of pillars, the stiffnesses of which are therefore to be added.

$$S_p \text{ (upper length)} = \frac{7{,}090}{12\tfrac{1}{2} \times 12} = 47$$

$$S_p \text{ (lower length)} = \frac{34{,}000}{12\tfrac{1}{2} \times 12} = 226$$

$$S_p \qquad \text{(total)} \qquad = 273$$

34,000 being the moment of inertia of the 24-in. square standard pillar (see *R.C.S.E.*, p. 53).

Therefore
$$\frac{F_p \,.\, S_p}{S_b} = \frac{4 \times 273}{\frac{78{,}000}{300}}$$
$$= 4{\cdot}25$$

whence the bending moment in the pillar is about $\frac{Wl}{25}$ (*R.C.S.E.*, p. 55).

$$B = \frac{Wl}{25} = \frac{83{,}333 \times 300}{25} = 1{,}000{,}000 \text{ in. lb.}$$

This again needs multiplying by $\frac{12}{9}$ to allow for third-point loading, and gives $B = 1{,}333{,}000$ in. lb. This moment needs to be divided between the upper and lower lengths of pillar in proportion to their stiffnesses (S_p), that in the lower length being

$$1{,}330{,}000 \times \frac{2{,}733}{3{,}300} = 1{,}092{,}000 \text{ in. lb.}$$

and the stress due to it being

$$C = \frac{Bd}{2I} = \frac{1{,}092{,}000 \times 24 \text{ in.}}{2 \times 34{,}000} = 386 \text{ lb./in.}^2$$

This again may be reduced for depth of beam and haunch to $C = 386 \times \frac{48 \text{ in.}}{75 \text{ in.}} = 247$ (see Appendix and Fig. 12).

Whence maximum stress is

$C = 470$ direct $+ 247$ bending $= 717$ lb./in.^{2} total.

It will be seen from these calculations that the interior columns are stressed to not greatly different figures whether they are fully loaded (in which case there is no bending

in them) or loaded from one side only, in which case the load is eccentric and produces bending stresses (the latter giving slightly greater results). It may, in fact, be conceded that, in general, interior columns need only be designed for the simple case of full load and concentric stress distribution when the arrangement is a symmetrical one. Nevertheless, the calculation was thought to be worth making in this instance to exemplify the correct method and to illustrate this fact.

It must also be remembered that the writer is disposed to allow a considerable raising of permissible stresses when secondary bending is fully taken into account (see *R.C.S.E.*, par. 72, pp. 58–9).

In outside pillars, however, the case is quite different. Bending occurs when the pillar is fully loaded, and the bending stress is not therefore offset by a corresponding reduction in direct stress. The bending stresses are also much higher. For both these reasons, it is essential that allowance for bending be made for outside pillars and that such pillars be made larger than if designed for direct stress only.

Column Type (*b*)

In the previous section we showed that standard columns 16 in. by 16 in. for the upper length and 24 in. by 24 in. for the lower lengths were stressed to about 600 lb./in.^{2}, whether we considered them fully loaded from all sides or whether we considered them loaded on one side only, when we found a smaller load, but a stress due to bending owing to eccentricity of loading. We concluded that as a rule bending need not be calculated for interior columns.

This is very different in the case of external columns. External columns get their worst load and bending due to eccentric loading simultaneously, and it therefore becomes essential to consider the stresses due to such bending.

We will calculate column type (*b*), being wall columns carrying one main beam and two secondary beams (Fig. 11A).

The load on wall beams 1 and 2 is as follows :

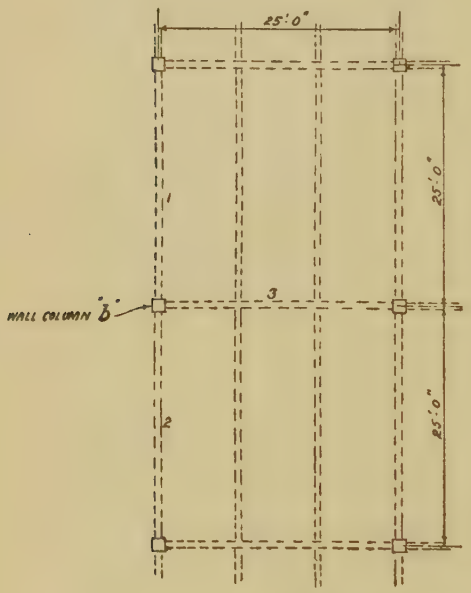
Floor load 4 ft. × 25 ft. × 300 lb. .	. = 30,000 lb.
Wall load 3 ft. × 1 ft. × 150 × 25 ft.	. = 11,250 ,,
	41,250 ,,

The load on main beam 3 is

$$25 \text{ ft.} \times 25 \text{ ft.} \times \tfrac{2}{3} \times 300 = 125{,}000 \text{ lb.}$$

The reaction on the column is at each floor therefore 20,625 lb. from each secondary beam, and 62,500 lb. from the main beam.

The reactions from the secondary beams are, however, balanced and therefore produce no bending of the column,



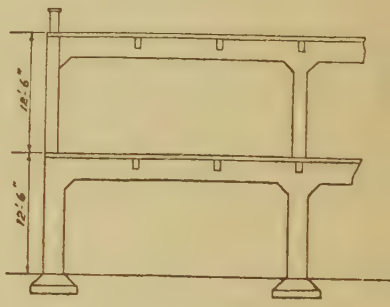


FIG. 11A.—Showing Wall Column *b*.

whereas that of the main beam clearly does produce bending.

In general the author recommends making wall columns the same size as interior columns, although they carry less load but more bending, until the fourth story from the top is reached.

Let us, in the present case, adopt standard 16-in. square columns for the upper tier and standard 24-in. square columns for the lower tier (see *R.C.S.E.*, p. 53, for standard columns), as we found required for the interior columns.

Then the direct stress in the upper tier is

20,625 lb. from beam 1.
20,625 ,, ,, 2.
62,500 ,, ,, 3.

103,750 lb.

$$c = \frac{103{,}750}{300} = 345 \text{ lb./in.}^2$$

300 being the equivalent area of the 16-in. square column (see *R.C.S.E.*, p. 53).

Similarly the direct stress in the lower tier is

$$c = \frac{207{,}500}{664} = 310 \text{ lb./in.}^2$$

664 being the equivalent area of the 24-in. column.

Upper Tier.—Following the calculation for stress due to bending [given in *R.C.S.E.*, Chap. V.] we have

$$F_p \text{ (outside pillars)} = 6$$

$$S_p = \frac{I}{l} \text{ of pillars}$$

$$= \frac{7{,}090}{150} = 47.$$

[7,090 being I for a 16-in. pillar (see *R.C.S.E.*, p. 53), 150 being the length of tier 12 ft. 6 in.]

$$S_b = \frac{I}{l} \text{ of beam}$$

$$= \frac{78{,}000}{300} = 260$$

Whence $$\frac{F_p \,.\, S_p}{S_b} = \frac{6 \times 47}{260} = 1{\cdot}09.$$

Whence bending moment in column is

$$\cdot 025Wl \times \frac{12}{9}$$

$$= \cdot 025 \times 125{,}000 \times 300 \times \frac{12}{9}$$

$$= 1{,}250{,}000 \text{ in. lb.}$$

[Note.—In the above, ·025 is obtained from Table II, *R.C.S.E.*, p. 54, W is the total load on the beam producing

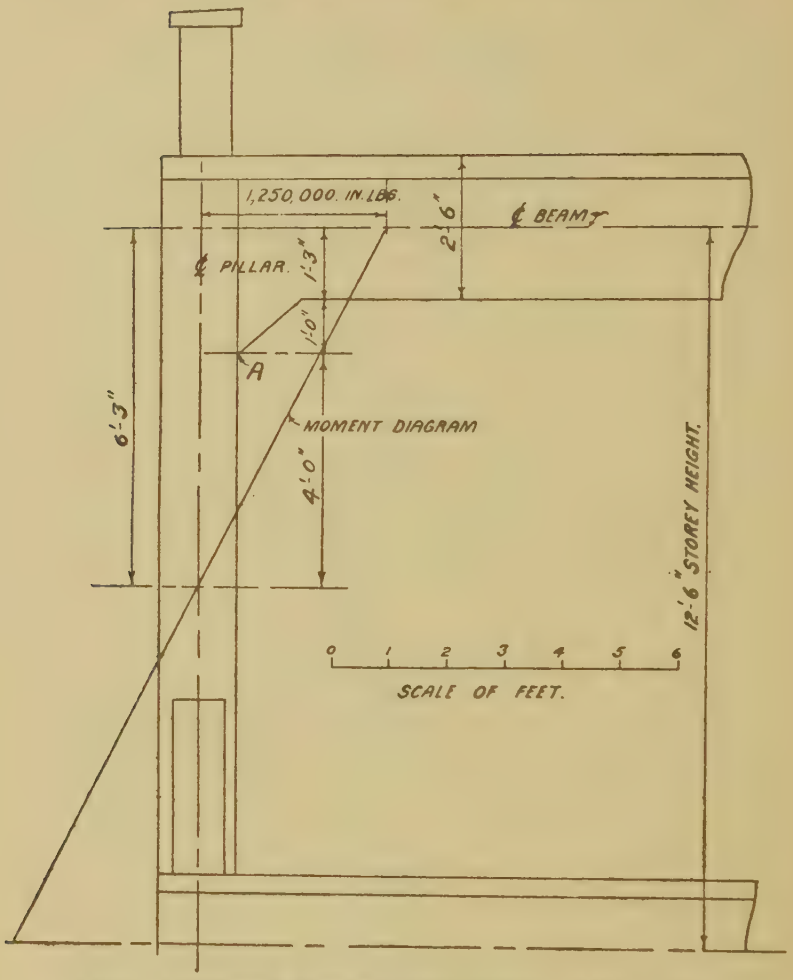


FIG. 12.—Moment diagram in Wall Column of Upper Tier.

bending in the column, l its span in inches, while $\frac{12}{9}$ is a coefficient required for third-point loading.]

The stress due to bending is

$$c = \frac{Bd}{2I} \text{ (see } \textit{R.C.S.E.}\text{, p. 55)}$$

$$= \frac{1{,}250{,}000 \times 16 \text{ in.}}{2 \times 7{,}090} = 1{,}400 \text{ lb./in.}^2$$

This stress may be reduced for the reduction in moment which takes place between the centre line of beam and the point *A* (see Fig. 12), which is clearly the point of worst stress, since above this the section increases rapidly.

With this correction, we may take the stress due to bending as

$$c = 1{,}400 \times \frac{4}{6\frac{1}{4}} = 900 \text{ lb./in.}^2$$

and the maximum stress as

$$\begin{aligned} c &= 345 \text{ direct} + 900 \text{ bending} \\ &= 1{,}245 \text{ lb./in.}^2 \end{aligned}$$

The author would not consider such a stress dangerous, provided a 1 : 1 : 2 concrete were used, since when bending is taken into account it is clearly absurd to ask for the same factor of safety as when it was ignored.

Lower Tier.—The only unbalanced load is the main beam reaction of 62,500 lb. as before. The bending in the column due to it is resisted by both the upper and lower tiers of column, as follows :—

$$F_p S_p \text{ (upper length)} = 6 \times \frac{7{,}090}{150} = 282$$

$$F_p S_p \text{ (lower length)} = 4 \times \frac{34{,}000}{150} = 904$$

$$F_p S_p \text{ total} = 1{,}186$$

The value of F_p for the upper length is taken as 6 because the top beam is deflected in the same way as the lower beam, while F_p for the lower length is taken as 4 because the footing is assumed horizontal, or undeflected by the bending of the beams.

$$S_b \text{ as before is } \frac{7{,}800}{300} = 260$$

whence

$$\frac{F_p \,.\, S_p}{S_b} = \frac{1{,}186}{260} = 4{\cdot}5.$$

Total moment in column $= {\cdot}053 \times 125{,}000 \times 300 \times \frac{12}{9}$

$= 2{,}650{,}000$ in. lb.

[Note ·053 is from Table II, *R.C.S.E.*, p. 54.

W = total load on beam bending the columns.

l = span in inches of this beam.

$\frac{12}{9}$ = factor to allow of third-point loading as before.]

Of this $2{,}650{,}000 \times \frac{904}{1{,}186} = 2{,}600{,}000$ in. lb. is carried on the lower tier column just under the lower floor, producing a stress of

$$c = \frac{Bd}{2I} \text{ (see } R.C.S.E., \text{ p. 55)}$$

$$= \frac{2{,}060{,}000 \times 24 \text{ in.}}{2 \times 34{,}000} = 720 \text{ lb./in.}^2$$

This can be reduced for depth of beam and haunch as before (see Fig. 13).

$$\text{Stress at } A = 720 \times \frac{8 \text{ ft. } 4 \text{ in.} - 2 \text{ ft. } 3 \text{ in.}}{8 \text{ ft. } 4 \text{ in.}}$$

$$= 720 \times \frac{73}{100} = 520 \text{ lb./in.}^2$$

whence max. stress

$c = 300$ lb./in.^{2} direct $+ 520$ lb./in.^{2} due to bending.
$= 820$ lb./in.^{2} total.

Note.—The moment diagram in Fig. 13, in which the moment at the bottom is half that at the beam level, assumes that the foundation remains fixed. (See *R.C.D.*, vol. i, p. 70).

Column Type (*c*)

Columns type (*c*) are wall columns carrying two main beams and one secondary beam. The calculation for this is on exactly similar lines to type (*b*), and therefore no useful purpose will be served here by making it.

The total load is practically the same, while the bending is less, since the two main beams 1 and 2 are balanced about the column and only the secondary beam 4 produces bending. Hence the stresses will be less than for columns (*b*) if the same sections are retained. In the calculations for bending, the factor $\frac{12}{9}$ in the last case will not occur as the load on the secondary beam is a distributed one, and not two-point loading. An interesting point is whether beams 3 and 5, which clearly also deflect, produce bending on the column. Clearly if main beams 1 and 2 had great torsional strength and torsional stiffness or rigidity, such bending would be transmitted to the column, but in most cases the torsional stiffness of the

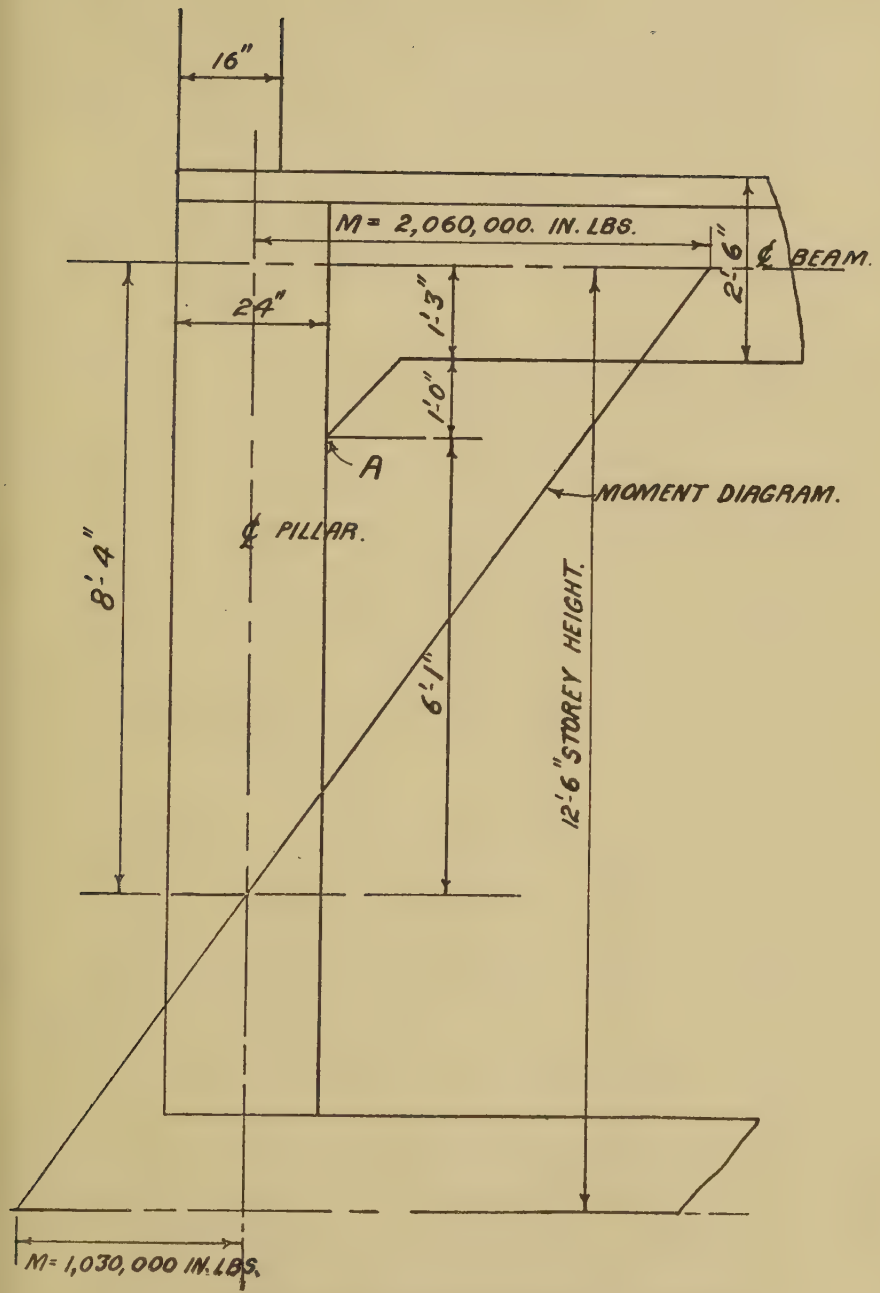


FIG. 13.—Moment diagram in Wall Column of Lower Tier.

main beam is so small compared with the bending stiffness that the effect of the bending of beams 3 and 5 is quite negligible on the column.

Column Type (*d*)

Type (*d*) are the four corner columns in the building. Each of these carries one external main beam (2), and

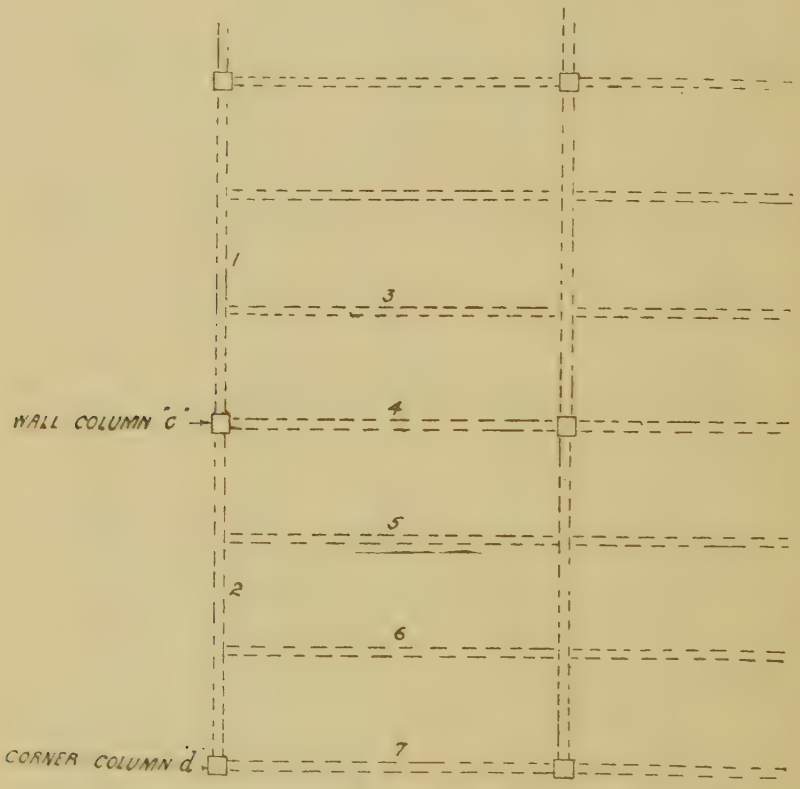


Fig. 14.—Showing Wall Column *c* and Corner Column *d*.

one external secondary beam (7). The loads on these (from previous calculations) are 73,700 lb. and 41,250 lb. respectively, making the column load at each floor half these figures:

$$36{,}850 + 20{,}625 = 57{,}475 \text{ lb.}$$

This example is worth working out, because this column

is subject to bending in two planes at right angles by the deflection of beams 2 and 7 respectively.

The direct stresses are

$$\frac{57{,}475}{300} = 192 \text{ lb./in.}^2 \text{ in the upper tier}$$

[57,475 being the load in lb. and 300 the equivalent area of the standard 16-in. column]

and $$\frac{114{,}950}{664} = 173 \text{ lb./in.}^2 \text{ in the lower tier}$$

[114,950 being the load in lb., and 664 the equivalent area of the standard 24-in. column].

Upper Tier.—After what has been done, the calculations need less explanation. But the wall beams, being rectangular beams above the slab forming a parapet on the upper floor and a wall under the window on the lower floor, are not standard T-beams, and therefore their moment of inertia I cannot be taken from the Table on p. 35 (*R.C.S.E.*), but needs to be calculated.

The main wall beam was described in the section on Main Beams in Walls as being 36 in. by 12 in. above a 5-in. slab ($h = 41$ in.) and having six 1-in. rods top and bottom and a resisting moment of 2,360,000 in. lb.

The most convenient method of calculating the moment of inertia in such a case is

$$I = \frac{Ry}{f}$$

where R is the resisting moment,

y is the distance of any point from the neutral axis,

f is the concrete stress at that point or the steel stress divided by 15.

Applying this in the present case and considering the top fibre, this is at a distance of $n = \cdot 36d = \cdot 36 \times 38 = 13 \cdot 7$ in. from the neutral axis.

$$\text{Whence } I = \frac{2{,}360{,}000 \times 13 \cdot 7}{600} = 54{,}000 \text{ in. units.}$$

Alternatively, calculating it from the steel in tension, its distance from the neutral axis is

$$d - n = 38 \text{ in.} - 13 \cdot 7 \text{ in.} = 24 \cdot 3 \text{ in.}$$

$$\text{whence } I = \frac{2{,}360{,}000 \times 24 \cdot 3 \times 15}{16{,}000}$$

$$= 54{,}000 \text{ in. units.}$$

The secondary wall beam would be the same size, for

general convenience, but less steel would be sufficient, and we will assume for our calculation that it has a moment of inertia of

$$I = 27{,}000 \text{ in. units.}$$

Bending due to Main Wall Beam.—

$$S_p = \frac{I}{l} \text{ of pillar (upper tier 16-in. standard pillar)}$$

$$= \frac{7{,}090}{150} = 47 \text{ as before.}$$

$$S_b = \frac{54{,}000}{300} = 180$$

[54,000 being I of wall beam,
300 being its span in inches],
F_p for outside pillars being 6,

whence

$$\frac{F_p \,.\, S_p}{S_b} = \frac{6 \times 47}{180} = 1{\cdot}57,$$

whence bending in column

$$B = {\cdot}03 \times 73{,}700 \times 300 \times \frac{12}{9}$$

$$= 880{,}000 \text{ in. lb.}$$

[·03 is from Table II, *R.C.S.E.*, p. 54.].

Stress in column due to bending

$$c = \frac{Bd}{2I}$$

$$= \frac{880{,}000 \times 16 \text{ in.}}{2 \times 7{,}090} = 1{,}000 \text{ lb./in.}^2$$

This can be reduced for depth of beam as before (though this time there is no haunch)

$$c = 1{,}000 \times \frac{6 \text{ ft. } 3 \text{ in.} - 1 \text{ ft. } 8 \text{ in.}}{6 \text{ ft. } 3 \text{ in.}}$$

$$= 1{,}000 \times \frac{53}{73} = 725 \text{ lb./in.}^2$$

Bending due to Secondary Wall Beam.—

$$S_p = 47 \text{ as before}$$

$$S_b = \frac{27{,}000}{300} = 90$$

$$F_p = 6$$

$$\frac{F_p \,.\, S_p}{S_b} = \frac{6 \times 47}{90} = 3{\cdot}14$$

$$B = {\cdot}04 \times 41{,}250 \times 300$$

$$= 500{,}000 \text{ in. lb.}$$

stress in column due to bending

$$c = \frac{Bd}{2I} = \frac{500{,}000 \times 16 \text{ in.}}{2 \times 7{,}090} = 564.$$

This can be reduced as before for depth of beam to

$$c = 564 \times \frac{53}{73} = 410 \text{ lb./in.}^2$$

Hence total maximum stress in column is

$c = 192$ lb./in.^{2} direct.
$+ 725$ lb./in.^{2} bending from main wall beam.
$+ 564$ lb./in.^{2} bending from secondary wall beam.

1,481 lb./in.^{2}

The student will now have no difficulty in calculating the stress in the lower tier, which will be less than the above.

The instructive thing which emerges from these calculations is that though all the columns are made the same size the stresses are least on the interior ones ($c = 688$), greater on wall columns type (b) ($c = 1{,}245$) and greatest on corner columns type (d) ($c = 1{,}481$), although the actual loads are least on the corner columns, greater on the wall columns, and greatest on the interior ones.

[Note.—The above stresses are for the upper tiers.]

Footings

If a square footing of length L carries a column of length of side l, the projecting parts of the footing will be subject to a bending moment which is greatest on the central section such as AA (see Fig. 15).

If we consider the total upward load on the left-hand side of this section, it is clearly $\frac{W}{2}$—where W is the total load on the footing—and similarly the total downward load on the left-hand side (occurring in the left-hand side of the column) is also $\frac{W}{2}$. If we assume constant stress distribution on the column and on the foundation soil, then the distances from the section AA (or centre line) of these upward and downward resultants are clearly $\frac{L}{4}$ and

$\frac{l}{4}$ respectively as shown in Fig. 15, whence the moment on AA is

$$B = \frac{W}{2} \times \frac{L}{4} - \frac{W}{2} \times \frac{l}{4}$$

$$\text{or } B = \frac{W}{8}(L - l).$$

This is a very simple and accurate formula, which enables us to calculate such footings with great ease.

Examples

The maximum load on interior columns was 375,000 lb. Assuming that the soil will safely carry 2 tons per sq. ft., the area of footing required would be

$$A = \frac{375{,}000}{2 \times 2{,}240} = 83{\cdot}2 \text{ sq. ft.}$$

whence $L = \sqrt{83{\cdot}2} = 9$ ft. approximately.

From previous calculations,

$$l = 24 \text{ in.}$$

$$\therefore B = \frac{W}{8}(L - l) = \frac{375{,}000}{8}(108 \text{ in.} - 24 \text{ in.})$$

$$= 3{,}950{,}000 \text{ in. lb.}$$

If we resist this by a rectangular slab 108 in. wide, reinforced with ·675 per cent. of steel, then the resistance moment will be

$$R = 95\, bd^2$$

$$= 3{,}950{,}000 \text{ in. lb.}$$

$$\text{whence } d = \sqrt{\frac{3{,}950{,}000}{95 \times 108}}$$

$$= 19{\cdot}6$$

and $h = 22$ in. to allow reasonable cover.

The area of steel required is given by—

$$A = \frac{{\cdot}675}{100}\, b.d$$

$$= \frac{{\cdot}675}{100} \times 108 \times 22$$

$$= 16 \text{ ins.}^2$$

Sixteen $1\frac{1}{8}$-in. diameter rods give the requisite area.

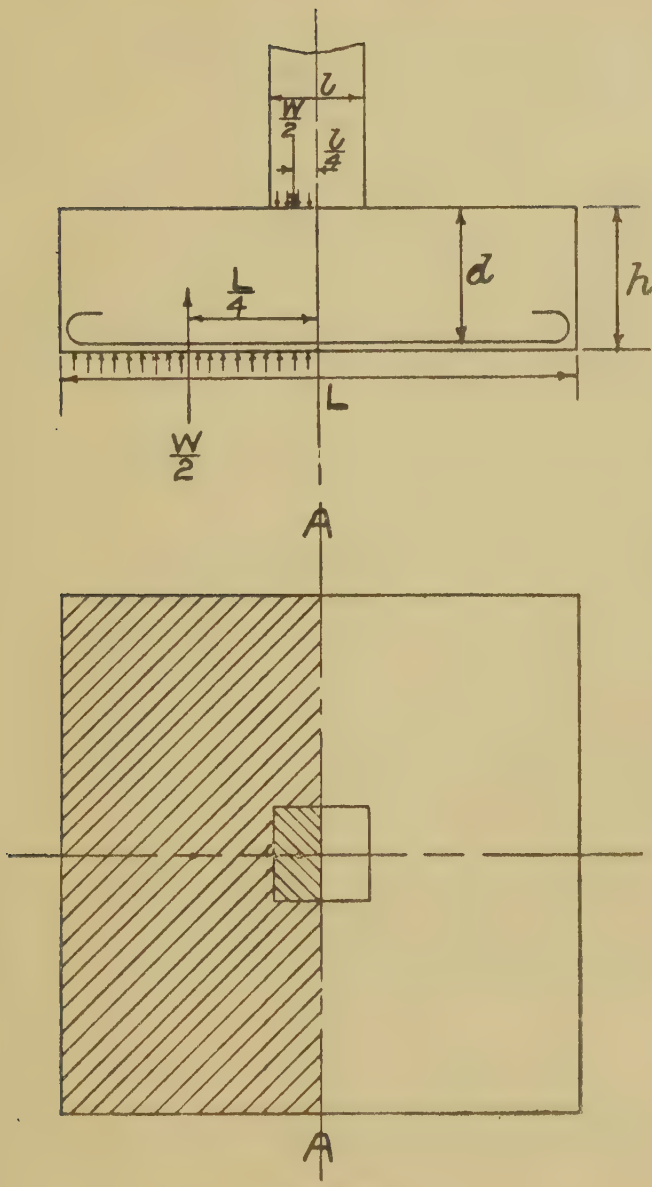


Fig. 15.

Now, clearly the centre section at right angles to *AA* would be subject to exactly the same moment, and therefore an additional sixteen $1\frac{1}{8}$-in. rods would be required in the other direction.

It will be unnecessary here to calculate the footings for the other columns, as they follow exactly the above example.

A few questions are worth discussing shortly, as they introduce points which an intelligent designer may raise.

First, as the columns may be subject to moment at the bottom under certain conditions, should not the footing also be so designed? The answer is, strictly speaking, yes; but an eccentricity which produces a large effect on a 24-in. column produces only a very small effect on a 9-ft. footing and may be neglected in most cases.

Second, is the footing illustrated the most economical type? The author finds it is for footings up to about 10 ft. square.

Some designers use a footing of pyramidical shape. The disadvantage of this is twofold. The sloping upper surface is either to be formed by centering, in which great additional cost is incurred which easily balances the cost of the concrete saved. Also this top centering tends to lift when the concrete is placed and has to be weighted down and is generally troublesome.

Sometimes it is attempted to construct the sloping concrete without centering by mixing it very dry. This frequently results in very poor honeycombed concrete and is a dangerous expedient.

The pyramidical form also requires to be much deeper, because the central section, instead of having the full width available for compression, has the two sides cut away on top just where the compression stress would occur, and a much narrower width has to be reckoned on.

EXAMPLE V

RETAINING WALL

RETAINING walls are of many types, according to whether the vertical wall is on the side of the base slab nearer the earth to be retained (as in the present case) or on the other side (in which case the base slab is covered with the full height of earth). The former kind requires a wider base, as it clearly is not so stable as when the base is heavily loaded with earth.

The pressure against a vertical face due to the weight of earth-fill against it depends greatly on conditions, and chiefly on the nature of the soil. A rock face may exert no pressure at all, while a quicksand may exert a pressure as high as

$$p = 100h$$

where p is the pressure in pounds per square foot and
h is the depth below the free surface in feet.

It will be noticed that this assumes a pressure distribution varying uniformly from zero at the top to a maximum near the bottom. The author considers this a closer approximation to the truth than the newer theory of Meem.

Actual soils lie between these two limits, and the experience of the engineer is called in here to determine a correct figure for an actual case.

We will assume that in our case this pressure can reasonably be taken at

$$p = 30h$$

which is a common figure for sound yellow clay and dry ballast.

If the free surface may be loaded (as by traffic, buildings, storage of goods, etc.) the front face will be subject to an additional pressure of

$p_1 = 30h_1$, where
h_1 is the height of earth,

which gives the same weight as the superload. In our

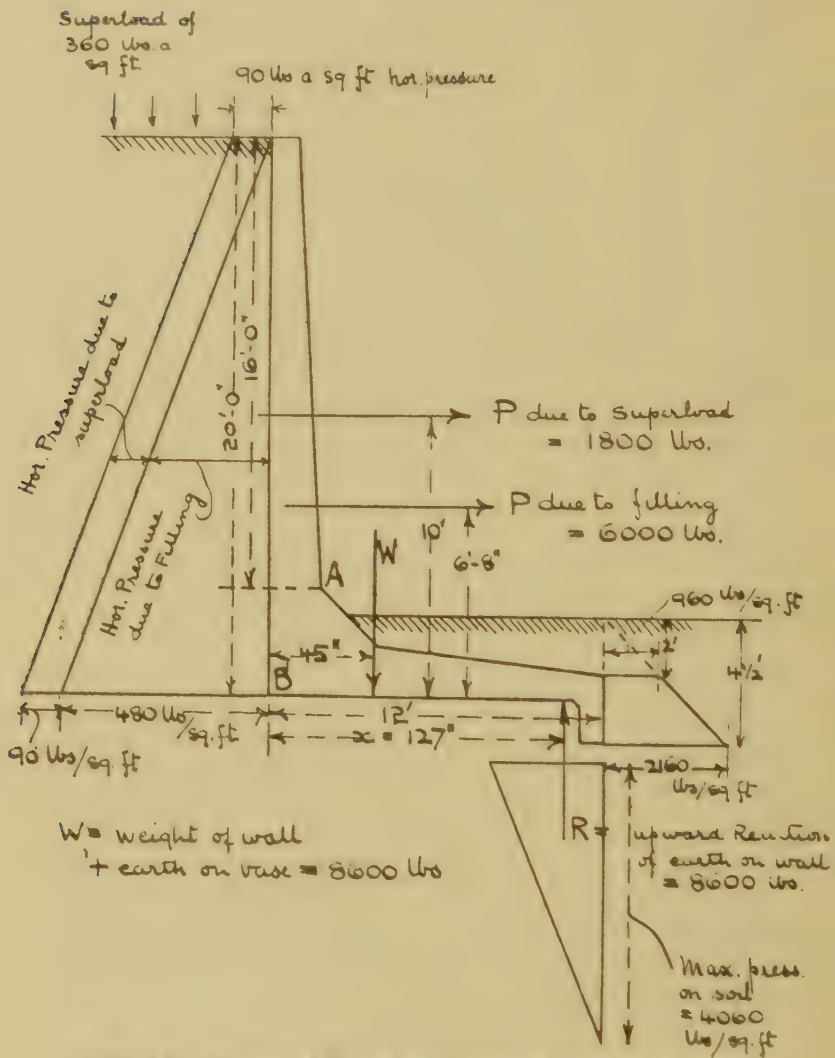


FIG. 16.—Diagram of Forces and Pressures Governing Stability.

case we will take the superload as 360 lb. sq. ft. (a common figure for a heavy floor or a street), whence

$$h_1 = \frac{360}{120} = 3 \text{ ft.}$$

[120 being the weight of the earth in lb./cu. ft.]

and $p_1 = 30 \times 3 = 90$ lb./sq. ft.

This additional pressure p_1 is a constant pressure right down the back of the wall.

Calculations of Strength.—The front wall may be considered as a cantilever fixed near the bottom, and loaded by these horizontal pressures.

The worst point is generally at the point A, at the top of the haunch, hence moments about this point should first be calculated as follows :—

[See Fig. 16 for pressure diagrams, etc.]

Due to Superload.—

Pressure due to superload=

$$30 \times 3 = 90 \text{ lb./sq. ft.}$$

Total pressure down to $A = 16 \times 90 = 1{,}440$ lb.

Moment about $A = 1{,}440 \times 96$ in. $= 138{,}000$ in. lb.

Due to Filling behind Wall.—Pressure varies from 0 at the top to

$$30 \times 16 = 480 \text{ lb./sq. ft.}$$

at the bottom.

Hence average pressure is 240 lb./sq. ft.

Total pressure above A is

$$16 \text{ ft.} \times 240 = 3{,}840 \text{ lb.}$$

This pressure has a triangular distribution, and therefore its resultant lies at a distance of

$$\frac{h}{3} = \frac{16 \times 12}{3} = 64 \text{ in.}$$

from A.

Hence moment about A

$$= 3{,}840 \times 64 \text{ in.} = 246{,}000 \text{ in. lb.}$$

Hence total moment about A

$$= 138{,}000 + 246{,}000$$
$$= 384{,}000 \text{ in. lb.}$$

Taking the resistance moment at

$$R = 95\, bd^2$$

and remembering that $b = 12$ in. [since we are considering a 12-in. width of wall] we have

$$d = \sqrt{\frac{384{,}000}{95 \times 12}}$$
$$= 18{\cdot}4 \text{ in.}$$

whence $h = 20$ in.

allowing a small cover of concrete.

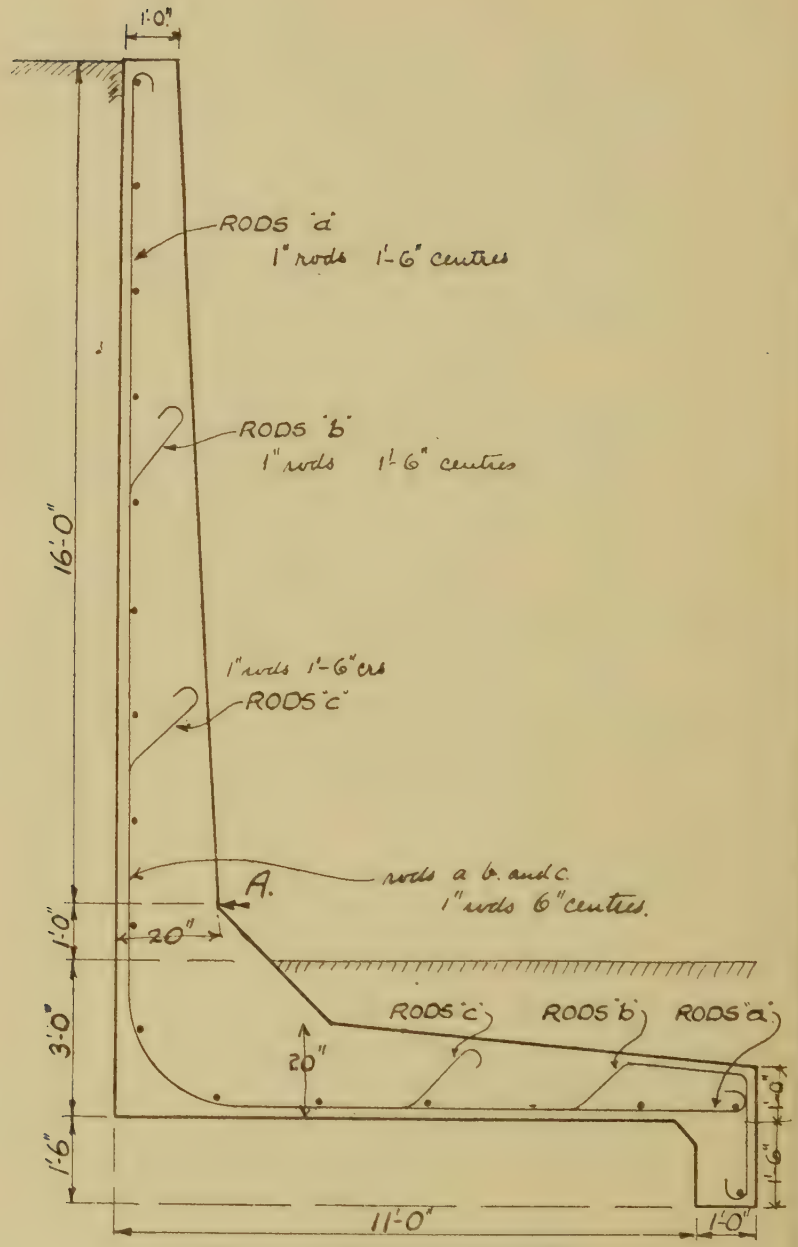


FIG. 17.—Design for Retaining Wall.

The formula $R = 95\ bd^2$ is based on a percentage of 0·675 of steel.

Whence the area of steel per foot run of wall required is

$$A = \frac{0{\cdot}675}{100} \times 12 \times 18{\cdot}4$$
$$= 1{\cdot}5 \text{ in.}^2$$

Hence 1-in. diameter rods at 6-in. centres give what is required.

It is not necessary for all these to run to the end, as the moment decreases rapidly as the top of the wall is approached, as may be seen by a trial further up.

The best arrangement in the present case is to divide the rods into three kinds, (*a*), (*b*), and (*c*), bent as shown in Fig. 17 and placed alternately. This will give one-third of the rods going to the top, two-thirds some two-thirds of the way up, and all the rods for about the lower third.

Shear.—The shear across *A* is clearly 1,440 + 3,840 = 5,280 lb. The shear resistance is

$$60 \times 18{\cdot}4 \times 12 = 13{,}200 \text{ lb.}$$

so that no question as to this arises.

This is practically always the case with walls of this type.

Other Sections.—The student would do well to repeat the moment calculations for other sections.

Stability.—As has been mentioned, strength is one thing, stability another. The latter depends on whether the resultant of all the forces (weight and pressure) lies sufficiently inside the base to give reasonable pressures on the soil.

In the present case, the weight may be calculated approximately as follows :—

Wt. of front wall per ft. run

$$= 20 \text{ ft.} \times 1\tfrac{1}{2} \text{ ft.} \times 150 = 4{,}500 \text{ lb.}$$

Wt. of base slab and earth on it

$$= 10\tfrac{1}{2} \text{ ft.} \times 3 \text{ ft.} \times 130 = 4{,}170 \text{ lb.}$$

The centres of gravity of these lie respectively about 9 in. and 81 in. from the point B. Hence the centre of gravity of the whole lies about 45 in. from B.

For stability we must now consider the horizontal pressures on the whole front face of the wall 20 ft. deep.

Due to Superload.—Pressure due to superload

$= 90$ lb./sq. ft. as before.

Total pressure on front wall

$= 90 \times 20 = 1{,}800$ lb.

Moment about B $= 1{,}800 \times 120$ in. $= 216{,}000$ in. lb.

Due to Filling.—Pressure varies from 0 at the top to $30 \times 20 = 600$ at the bottom. Hence the average pressure is 300.

Total pressure above B is $20 \times 300 = 6{,}000$ lb.

$$\text{Moment about B} = \frac{6{,}000 \times 240 \text{ in.}}{3}$$

$= 480{,}000$ in. lb.

The total overturning moment about the point B is therefore as follows :—

Due to weight of wall 8,600 lb. $\times$ 45 in.	= 387,000	in. lb.
,, superload	216,000	,,
,, filling	480,000	,,
Total	1,083,000	,,

This has to be resisted by the contrary resisting moment due to the reaction on the soil.

This reaction has a vertical component equal to the weight of the wall

$$W = 8{,}600 \text{ lb.}$$

Hence the horizontal distance of this reaction from the point B must clearly be

$$x = \frac{M}{W}$$

$$= \frac{1{,}083{,}000}{8{,}600}$$

$= 127$ in.

This point is therefore $144 - 127 = 17$ in. from the right-hand side of the base.

Assuming a triangular distribution of the upward pressure of the soil on the base, whose resultant has to come at this point, it follows that the length of base subject to such upward pressure is

$$3 \times 17 = 51 \text{ in.}$$

Hence the average pressure on the soil over this distance is

$$p = \frac{W}{A} = \frac{8{,}600 \text{ lb.}}{\frac{51}{12} \text{ ft.}^2} = 2{,}030 \text{ lb./ft.}^2$$

The maximum pressure being twice the average in triangular distribution, the *maximum* pressure is

$$p = 4{,}060 \text{ lb./ft.}^2$$

or just under 2 tons a sq. ft., which is safe on most soils.

The above calculations will be clearer by reference to the diagram in Fig. 16. Notice that the base is $\frac{6}{10}$ the height of the wall and just sufficient for stability. Where the superload is relatively greater—in proportion to the earth pressure—this may require increasing; where it is negligible, the ratio may come down to $\frac{1}{2}$.

Where the vertical wall is at the other side of the horizontal base, so that the base is earth-covered to the full height, the ratio may be as low as $\frac{4}{10}$.

Sliding.—Walls sometimes fail by sliding and not by overturning, and care must be taken as regards this. The horizontal pressure is 1,440 + 3,840.

$$P = 5{,}280 \text{ lb.}$$

The vertical weight of wall is

$$W = 8{,}600 \text{ lb.}$$

If sliding is to be prevented by friction on the base, the coefficient of friction would require to be

$$\phi = \frac{5{,}280}{8{,}600} = 0{\cdot}62.$$

This is very high, and could hardly be relied on except on a rocky base with a rough surface. With ordinary earth foundations of dry clay, ballast, etc., a figure of 0·25 is generally safe.

[Note, however, that a *wet* clay surface may give much less and must be guarded against.]

In that case, the horizontal force resisted by friction is

$$\cdot 25 \times 8{,}600 = 2{,}150 \text{ lb.}$$

leaving 5,280 − 2,150 = 3,130 lb. To resist this, a heel beam has been provided in the design.

The safe horizontal pressure of such a surface against

the earth (before the earth pushes away) depends, of course, on the earth, but may (under average conditions of sound yellow clay or ballast) be taken as

$$p = 480h$$

where h is the depth in feet below the free surface and p the pressure in lb. per sq. ft.

In our case, h at the top of the heel beam is

$$h = 2$$

whence $p = 960$ lb./sq. ft.

At the bottom of the heel beam

$$h = 4\tfrac{1}{2}$$

$$p = 2{,}160 \text{ lb./sq. ft.}$$

Therefore the mean safe pressure is

$$p = \frac{960 + 2{,}160}{2} = \frac{3{,}120}{2}$$

$$= 1{,}560 \text{ lb./sq. ft.}$$

Hence the safe resistance is

$$1{,}560 \times 2\tfrac{1}{2} = 4{,}400 \text{ lb.}$$

[$2\frac{1}{2}$ being the sq. ft. of surface of the heel beam against the resisting earth].

As this exceeds our pressure of 3,130 lb. to be provided against, we are clearly on the safe side.

Practical Points.—See that the back of the wall is drained by filling loose rubble immediately behind and *adequate* weep holes near the bottom for egress of water.

See that the foundation is cleaned *dry* and loose mud removed before concreting the footings, to prevent sliding.

EXAMPLE VI

WATER TOWER

We will take as our example the water-tower illustrated in Fig. 18.

The main dimensions are 16 ft. by 16 ft. by 8 ft. inside the reservoir, and the bottom is 50 ft. above the foundations.

Allowing for 7 ft. depth of water, the capacity would be

16 ft. by 16 ft. by 7 ft. = 1,800 cu. ft.
or 11,200 galls.
which weighs 112,000 lb.

After what has already been said in the previous examples, and particularly in Example II, it will be assumed, for the sake of avoiding repetition, that the student can design the tank portion.

We will assume that the main dimensions resulting from such design are as follows :—

Roof	4 in. thick
Walls	8 ,,
Bottom	12 ,,

The weight of the tank may then be determined as follows :—

Roof and cornice	24 ft. by 24 ft. by 50 lb.	= 28,800 lb.
Bottom	18 ft. by 18 ft. by 150 lb.	= 48,600 ,,
Sides and curb	4 × 17 ft. by 9 ft. by 100 lb.	= 61,200 ,,
Beams and projections	*say,*	10,000 ,,
		148,600 ,,
	say,	150,000 ,,

The weight of the tower, assuming the columns or pillars to be 18 in. square and the braces 18 in. by 8 in., will be

4 pillars	4 × 50 ft. × $2\frac{1}{4}$ × 150 .	= 67,500 lb.
Upper braces	4 × 12 ft. × 150 . .	= 7,200 ,,
Lower braces	4 × 17 ft. × 150 . .	= 10,200 ,,
		84,900 ,,
	say,	85,000 ,,

The total load is therefore

Water	112,000 lb.
Tank	150,000 ,,
Tower	85,000 ,,
	347,000 ,,

Hence the load per pillar (with no wind) is

$$\frac{347{,}000}{4} = 86{,}750 \text{ lb.}$$

Taking our standard 18 in. by 18 in. pillar (see *R.C.S.E.*, p. 53) it will be seen that the standard reinforcement is eight $\frac{3}{4}$-in. rods, the equivalent area is 373 sq. in., and hence the stress is $\frac{86{,}750}{373} = 233$ lb./in.^{2}

It may be thought that this is rather low, but we will show that the action of wind is to increase this considerably.

In the above calculation, no correction has been made for the obliquity of the pillars. Strictly speaking, the above stress should be increased in the ratio of the oblique length of the pillar to its vertical height. For small obliquities, as in the present case, this effect is negligible.

The effect of wind is chiefly twofold:

(1) To move the resultant thrust on the ground from the centre of the tower towards the leeward pillars, increasing the load on these and reducing the load on the windward ones.

(2) To produce a racking effect on the tower, which is resisted by the stiffness of the joints, and produces moments in the pillars and braces.

The splayed form of tower as illustrated in this example is very helpful by reducing the stresses from both these causes.

We will first calculate the wind pressures as follows, taking the unit pressure as 30 lb./sq. ft. of exposed surface.

W_1 = Wind pressure on tank
= 18 ft. by 10 ft. × 30 lb. = 5,400 lb.

Its moment about the base is

$$M_1 = 5{,}400 \times 55 \text{ ft.} = 297{,}000 \text{ ft. lb.}$$

The area of tower (two pillars and two braces) in elevation is

$$2 \times 50 \times 1\tfrac{1}{2} + 30 \times 1\tfrac{1}{2} = 150 + 45 = 195 \text{ sq. ft.}$$

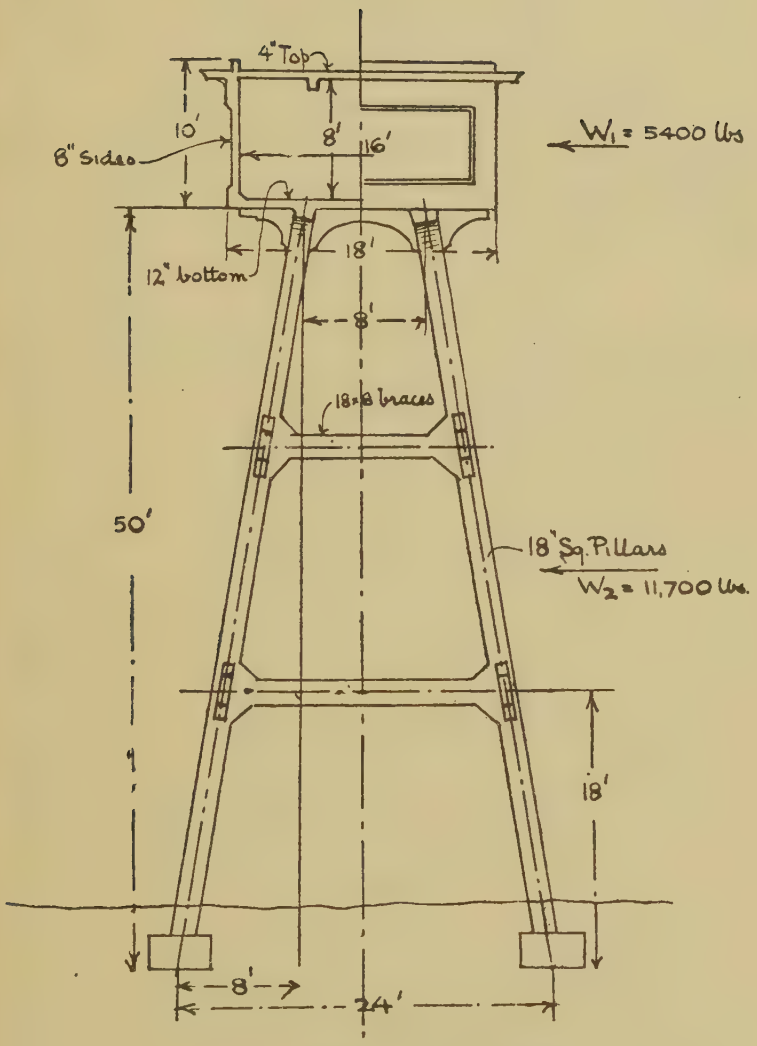


FIG. 18.—Diagram of Water Tower.

Actually, however, the wind will exert nearly the same pressure on the back pillars and braces, as the shielding effect of the front ones is negligible.

Hence W_2 = Wind pressure on tower
$= 2 \times 195 \times 30 = 11{,}700$ lb.

Its moment about the base is

$$M_2 = 11{,}700 \times 25 \text{ ft.} = 292{,}000 \text{ ft. lb.}$$

Hence the total wind moment about the base is $M_1 + M_2 = 589{,}000$ ft. lb.

This produces an additional pressure on the two leeward pillars of

$$\frac{589{,}000}{24} = 24{,}600 \text{ lb.}$$

or 12,300 lb. on each of them, and of course, reduces the load on the two windward pillars by an equal amount.

Hence the corrected direct stress in the leeward pillars after allowing for wind effect No. 1 is

$$\frac{86{,}750 + 12{,}300}{373} = \frac{99{,}050}{373} = 265 \text{ lb./in.}^2$$

Coming now to the effect of racking due to the wind, this is not quite so simple, but it is hoped that the following puts it as clearly as is possible in a small space.

Let us consider the lowest panel, i.e. under the lowest brace. This is clearly where the effect is most marked. The total horizontal wind force across this panel is clearly

$$W_1 + W_2 = 17{,}100 \text{ lb.}$$

If the pillars were vertical, the whole of this would produce racking. But the pillars being inclined as drawn, the loads in them have horizontal components. The horizontal component of the two leeward pillars is

$$2(86{,}750 + 12{,}300) \times \frac{8'}{50'} \text{ against the wind,}$$

and that of the two windward pillars is

$$2(86{,}750 - 12{,}300) \times \frac{8'}{50'} \text{ with the wind.}$$

It will be seen that the nett effect is to produce a total horizontal force of

$$4 \times 12{,}300 \times \frac{8'}{50'} = 7{,}850 \text{ lb.}$$

against the wind.

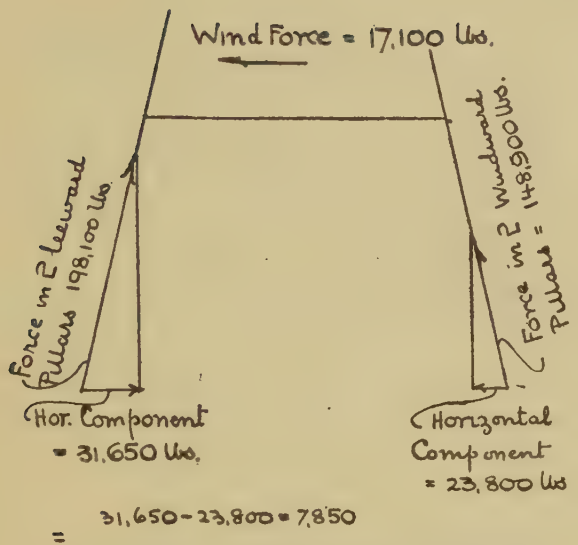

Wind Force = 17,100 lbs.
Force in 2 Leeward Pillars 198,100 lbs
Force in 2 Windward Pillars = 148,900 lbs.
Hor. Component = 31,650 lbs.
Horizontal Component = 23,800 lbs
31,650 - 23,800 = 7,850
=

FIG. 19.—Diagram of Forces in Pillars.

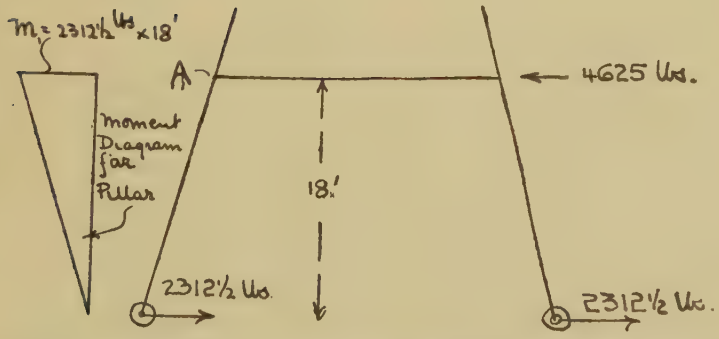


FIG. 20.—Forces on Each Pillar producing Moments (or Racking)—Pillars with Hinged Ends.

In other words, the wind force producing racking across the panel is reduced from 17,100 lb. to 17,100 − 7,850 = 9,250 lb.

This is a very valuable reduction and well illustrates the importance of inclined supports in structures of this kind, which was taken full advantage of in the Eiffel Tower.

This effect of the inclined pillars assisting in resisting wind is further illustrated in Fig. 19. It will be seen that the nett force helping to resist wind is

$$31{,}650 - 23{,}800 = 7{,}850 \text{ lb.}$$

as before. The remainder of the wind force, 9,250 lb., which is not taken up by the difference in the horizontal

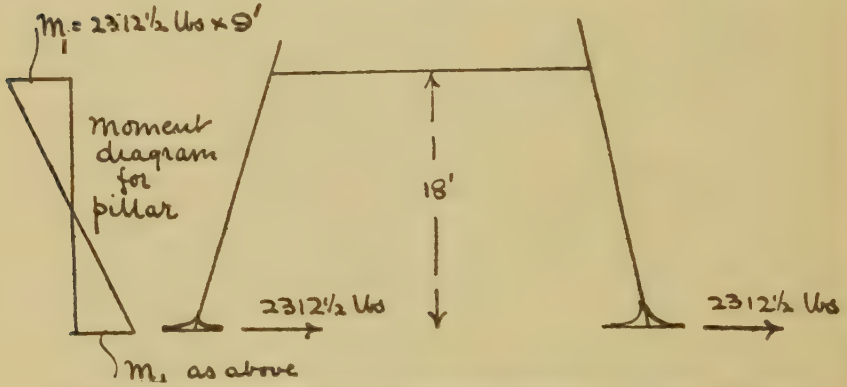


Fig. 21.—Forces on Each Pillar producing Moments (or Racking)—Pillars with Fixed Ends.

components in the pillar thrusts, clearly produces bending in the pillars.

This is, perhaps, made clearer in Fig. 20, where the force is shown as shared equally by the four pillars. If the bases are considered as hinged or rolling, then clearly these forces would produce a moment of $2{,}312\frac{1}{2} \times 18 = 41{,}600$ ft. lb. at the point A in the pillar.

If, however, as more generally is the case in good design, the base is made large enough so that it may be considered fixed in direction, then the moment may be considered as distributed as shown in Fig. 21 and to have half the previous value, namely,

$$M = 20{,}800 \text{ ft. lb.}$$
$$= 249{,}600 \text{ in. lb.}$$

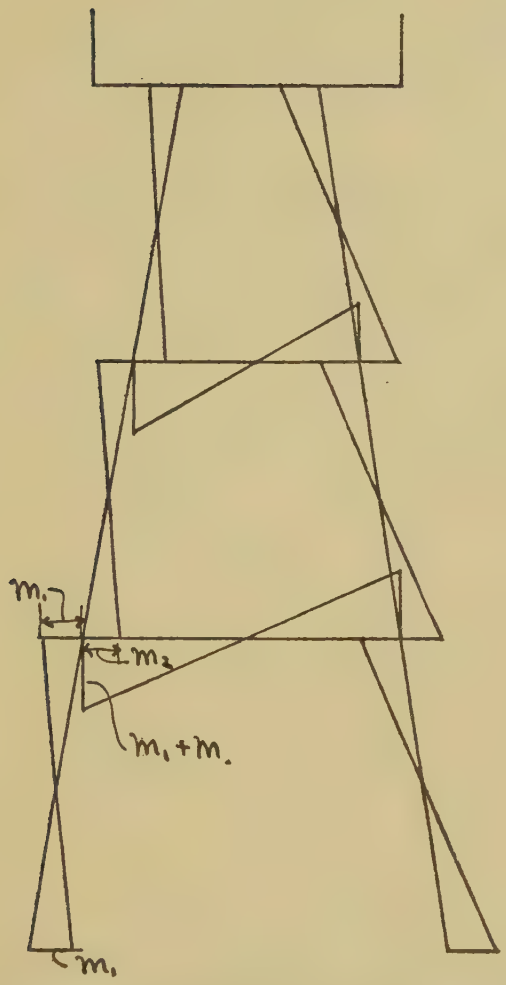


FIG. 22.—Diagram of Racking Moments in Pillars and Braces.

As the moment of inertia of our standard 18-in. by 18-in. pillar is 10,510-in. units (see *R.C.S.E.*, p. 53), the stress in it due to this moment would be

$$\frac{Md}{2I} = \frac{249{,}600 \times 18 \text{ in.}}{2 \times 10{,}510}$$
$$= 214 \text{ lb/in.}^2$$

Hence the maximum stress in the pillars is :

direct	265 lb./in.^{2}
bending or racking	214 ,,
Total	479 ,,

If we now consider the pillars in the panel above, we shall obtain very similar results, so that the moment diagram in the pillars is as indicated in Fig. 22. It will

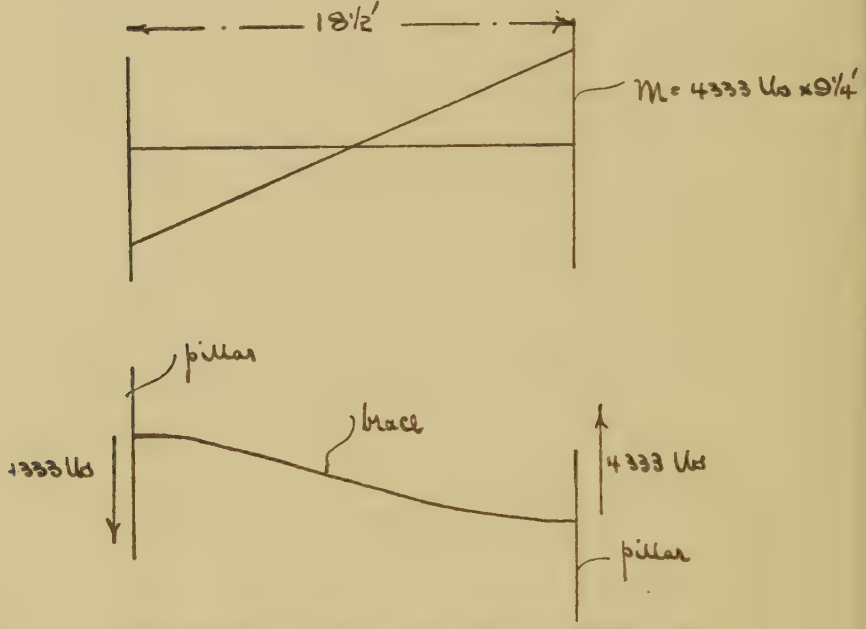


Fig. 23.—Moment Diagram and Racking Effect on Brace.

be clear that the end of the brace has to resist (because there is nothing else to do so) the moment M_1 from the pillar below plus the nearly equal moment M_2 from the pillar above. So that the moment in the brace will be roughly

$$M_1 + M_2 = 2M_1 \text{ approx.} = 499{,}200 \text{ in. lb. (approx.)}$$

at the ends, and varying along it as drawn.

This result can be obtained another way as a check. The effect of wind, as already stated, has been to add 12,300 lb. on to each of the two leeward pillars. This may be

considered as added in three approximately equal instalments at each of the panel points, namely, 4,333 lb. at each. The moment required at the fixed end of a brace $18\frac{1}{2}$ ft. long, to carry this load at the other end (being free), would clearly be

$$\begin{aligned} & 18\tfrac{1}{2} \text{ ft.} \times 4{,}333 \text{ lb.} \\ = \; & 80{,}000 \text{ ft. lb.} \\ = \; & 960{,}000 \text{ in. lb.} \end{aligned}$$

If both ends are equally fixed, this moment is shared between them, giving a moment of 480,000 in. lb. at each end. This agrees with sufficient accuracy with the previous result.

A diagram illustrating this method of treating the braces is given in Fig. 23.

If we make the braces 18 in. by 8 in., the steel required at the end would be about

$$\frac{480{,}000}{14 \text{ in.} \times 16{,}000} = 2{\cdot}15 \text{ in.}^2$$

say, two $1\frac{1}{8}$-in. rods top and bottom.

In addition, haunches and haunch bars are desirable at the ends, so as to provide adequate adhesion.

APPENDIX

NOTES ON MOMENTS IN PILLARS

Notes on Moments in Pillars due to Deflection of Beams Monolithic therewith.—The bending of pillars due to the deflection of beams monolithic with them has been touched upon in *R.C.S.E.*, Chap. V., but it is felt that a further explanation is desirable.

Case I.—If a pillar, originally vertical and straight, with one end (i.e. the lower one) held in position but not in direction by a form of fixing approximating to a hinge, has the other end (i.e. the upper one) deflected through a certain angle a (see Fig. 24), then it can be shown that the bending moment in the pillar at the deflected end can be expressed by the formula

$$B = 3\frac{I}{l}Ea \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where I = Moment of inertia of pillar.
l = Length of pillar.
E = Young's modulus of the material of the pillar.
a = Angle of deflection.

The distribution of bending moment is given by Fig. 24 (b), showing that it varies from a maximum at the bent end to zero at the free end.

If for $\frac{I}{l}$ we substitute S_p, the stiffness factor of the pillar, the expression may be written

$$B = 3S_pEa \quad . \quad . \quad . \quad . \quad . \quad (1a)$$

Note, in passing, that, except for the factor 3, it is obvious, with a little consideration, that the moment must vary with the quantities in the formula. One's personal experience with common objects should make the following remarks obvious.

Let us consider them in order.

The moment of inertia (which, for a plain rectangular section is given by $I = \frac{bd^3}{12}$; where b is the breadth and d the depth of the section) is a measure of the stiffness of the section, and obviously the stiffer the section, the greater will be the moment necessary to deflect the end through a given angle.

The length, on the other hand, makes for flexibility, which explains why the moment varies *inversely* with the

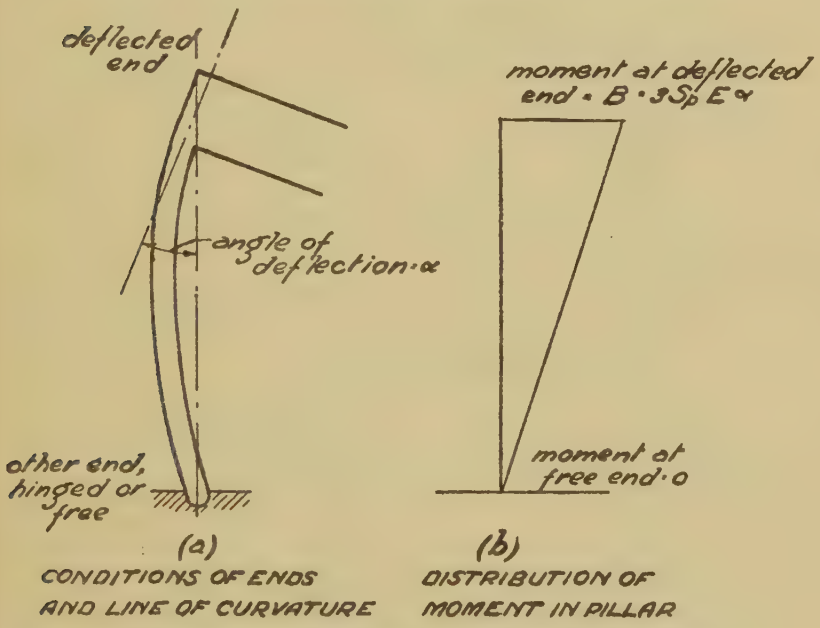

deflected end
angle of deflection · α
other end, hinged or free
(a)
CONDITIONS OF ENDS AND LINE OF CURVATURE
moment at deflected end · B · 3Sp E α
moment at free end · 0
(b)
DISTRIBUTION OF MOMENT IN PILLAR

Fig. 24.—Pillar with one end deflected and other end free.

length. Clearly a long pillar of a given section is more easily deflected through a given angle than a short one.

Young's modulus (see *R.C.S.E.*, par. 7) is a measure of the stress required to produce a given strain or change of shape, and clearly it requires a larger moment to deflect a pillar of a high modulus (such as steel) than one of the same section but of a low modulus (such as indiarubber) through the same angle.

The remaining expression is the angle through which one end of the pillar is deflected, and here again it is

clearly common experience that it requires a larger moment to deflect the pillar through a large angle than a small one.

The author would remark in passing that it is very important, in his view, to *realise* the meaning of a formula in this way, so that instead of representing so many dead algebraic symbols, it may become alive with the truth of common experience, when it loses its terrors and becomes a simple statement of a simple fact which can be tested by anyone with sufficient energy and interest to desire it.

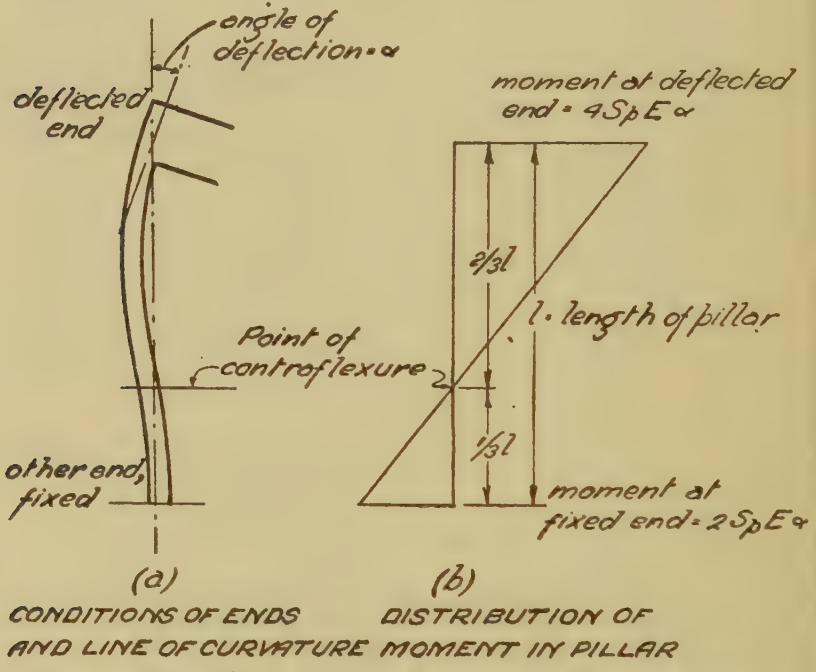


FIG. 25.—Pillar with one end deflected and other end fixed in direction.

Case II.—If, now, we consider a pillar having its bottom end fixed in position and direction, and its upper end deflected through a certain angle a, then it can be shown that the moment which has to be exerted at the top may be expressed by

$$B = 4S_pEa \quad . \quad . \quad . \quad . \quad (2)$$

and the distribution of moment is given in Fig. 25 (b), which shows that there is a moment of $2S_pEa$ at the restrained end, and a point one-third the length therefrom

where the moment is zero and the curvature changes (called a point of contraflexure).

Case III.—If a pillar originally straight and vertical has both ends deflected through an angle *a in the same*

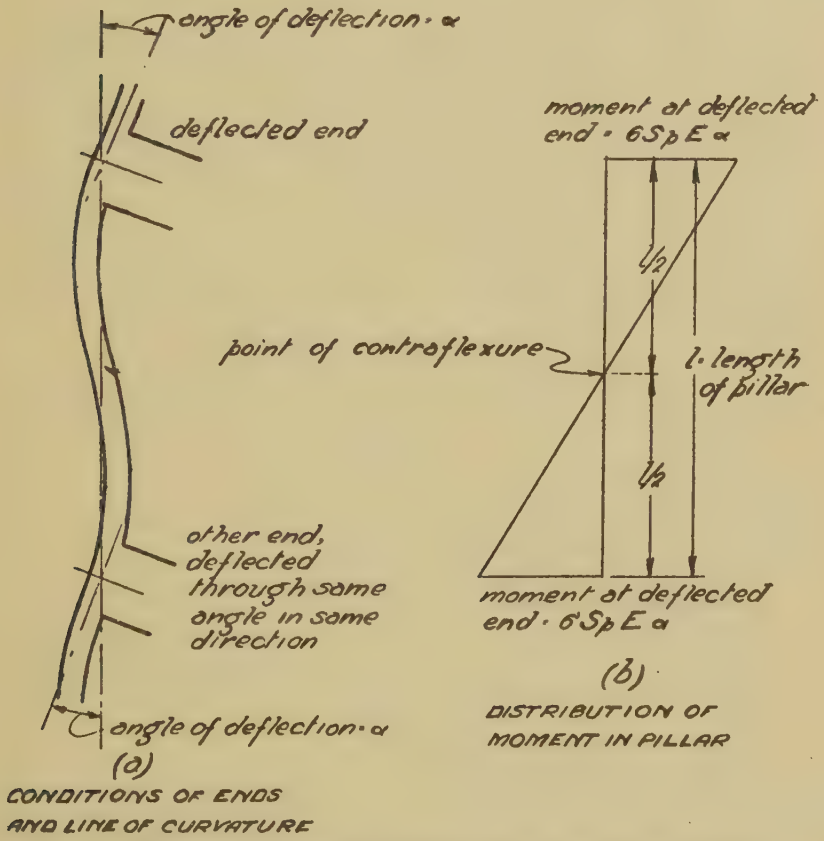

angle of deflection · α
deflected end
other end, deflected through same angle in same direction
angle of deflection · α
(a)
CONDITIONS OF ENDS AND LINE OF CURVATURE
moment at deflected end · 6SpEα
l/2
point of contraflexure
l · length of pillar
l/2
moment at deflected end · 6SpEα
(b)
DISTRIBUTION OF MOMENT IN PILLAR

Fig. 26.—Pillar with both ends deflected through same angle in same direction.

direction (see Fig. 26) it can be shown that the moment at the ends is given by

$$B = 6S_pEa \quad . \quad . \quad . \quad . \quad (3)$$

and the distribution of moment is as in Fig. 26 (*b*).

Case IV.—If a pillar, originally straight and vertical, has both ends deflected through an angle *a in opposite*

directions (see Fig. 27), it can be shown that the moment at the ends (and in this case throughout the length) is given by

$$B = 2S_pEa \quad . \quad . \quad . \quad . \quad (4)$$

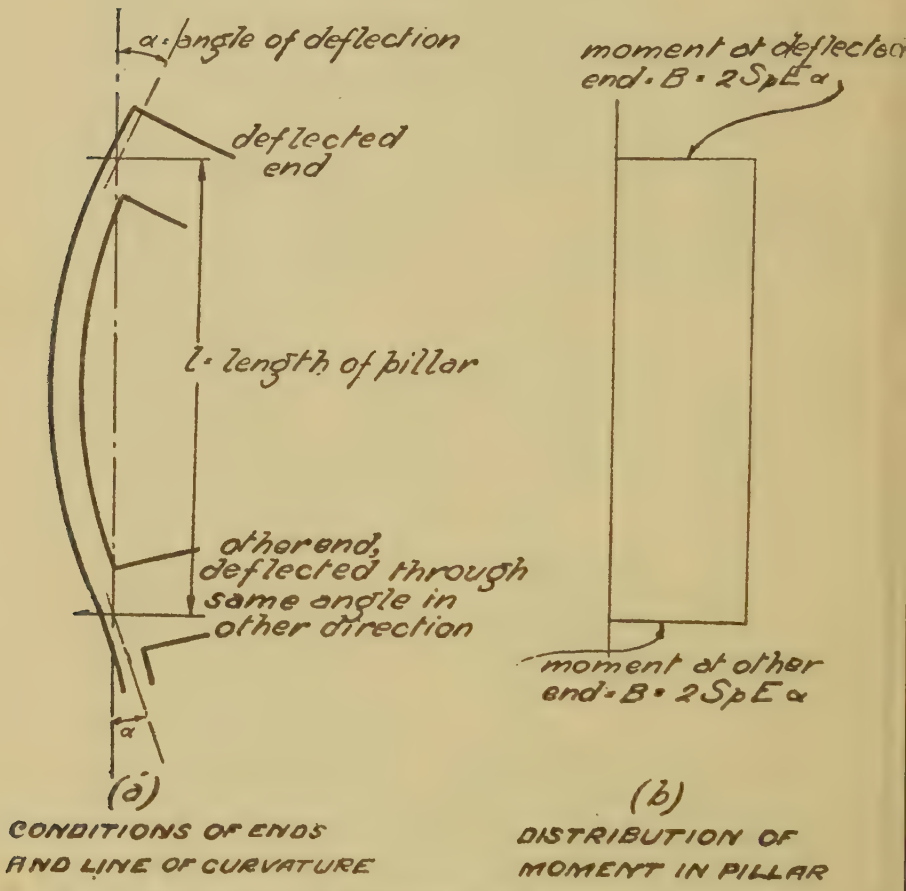

α · angle of deflection
deflected end
l · length of pillar
other end, deflected through same angle in other direction
α
(a)
CONDITIONS OF ENDS AND LINE OF CURVATURE
moment of deflection end · B · 2SpEα
moment at other end · B · 2SpEα
(b)
DISTRIBUTION OF MOMENT IN PILLAR

FIG. 27.—Pillar with one end deflected and other end deflected through same angle in other direction.

Now each of these cases approximates in some degree to pillars under practical conditions, the deflection of the end being produced by that of a beam under load, made monolithic with the pillar.

Case I approximates to the case of a pillar having its upper end monolithic with a flexible beam and its lower

end free to rotate (as for example if it rests on a small footing on yielding subsoil).

Case II approximates to that of a pillar with its upper end monolithic with a flexible beam and its lower end fixed (as, for example, if it rests on a large footing on a subsoil free from yield, or is attached here to a very stiff beam and pillar).

Case III approximates to a pillar having both ends monolithic with floor beams on two floors which are so loaded as to deflect the ends equally in the same directions.

Case IV approximates to a pillar having both ends monolithic with floor beams in two floors which are so loaded as to deflect the ends equally but in opposite directions.

It will be seen that in all cases the moment in the pillar may be written in the form

$$B = F_p S_p E a \quad . \quad . \quad . \quad . \quad . \quad (5)$$

where F_p is a factor, called the *Fixity* factor (because it depends on the condition of *Fixity of the Pillar*) and which has the value of

3 in Case I.
4 in Case II.
6 in Case III.
2 in Case IV.

Now in practice it is unusual to find a case *exactly* represented by one of the above. Most cases lie intermediate. Cases do occur, however, in which F_p may have a higher value even than 6.

The *exact* determination in any given case lies rather outside the scope of a simple exposition, and it must suffice here that Case II ($F_p = 4$) may be taken as a normal one for internal pillars, and Case III ($F_p = 6$) as a normal one for outside pillars.

The value of the moment in the pillar is given by the Tables in *R.C.S.E.*, pp. 54 and 55. It might be thought at first sight that it could be obtained direct from one of the formulæ (1) to (4) above, but these involve the expression a, the angle of deflection of one end of the pillar. This angle depends not only on the stiffness of the pillar but also on that of the beam. Clearly, if the beam were infinitely stiff, the angle of deflection would be zero.

It is therefore natural to expect this angle, and therefore the moment in the pillar (which depends on the angle), to depend on an expression involving the stiffness of the beam as well as that of the pillar, and reference to Tables II and III (see *R.C.S.E.*, pp. 54 and 55) shows that it depends on the expression

$$\frac{F_p S_p}{S_b},$$

where F_p is the fixity factor (2 to 6 as just explained)

S_p is the stiffness factor of the pillar, $\frac{I}{l}$,

S_b is the stiffness factor of the beam, $\frac{I}{l}$.

It will be noticed that in both Tables the moment varies from $\frac{Wl}{12}$ when $\frac{F_p S_p}{S_b}$ is infinite, to zero when $\frac{F_p S_p}{S_b}$ is zero (which are the two extremes between which all practical cases lie). This again is reasonable and obvious with a little consideration.

Let us consider the case when $\frac{F_p S_p}{S_b}$ is infinite. This means that the pillar is very stiff indeed in comparison with the beam. Under these circumstances, any attempt to deflect the beam by loading it will be resisted by a very stiff pillar, so that for all practical purposes the end of the beam is fixed in direction, and it is well known that a beam uniformly loaded and having its ends fixed in direction will have a moment equal to

$$B = \frac{Wl}{12}$$

at the ends.

Consider now the case when $\frac{F_p S_p}{S_b}$ approximates to zero. This means that the beam is very stiff indeed in comparison with the pillar. Hence the deflection will be negligible, and as the pillar is very flexible, a negligible deflection will exert only a negligible moment in it, when B will approximate to zero.

In practice, the end of a pillar generally enlarges into a beam section of appreciable depth.

The end of the pillar may then generally be taken at a point half-way up the height of the beam.

The worst straining action on the pillar then occurs

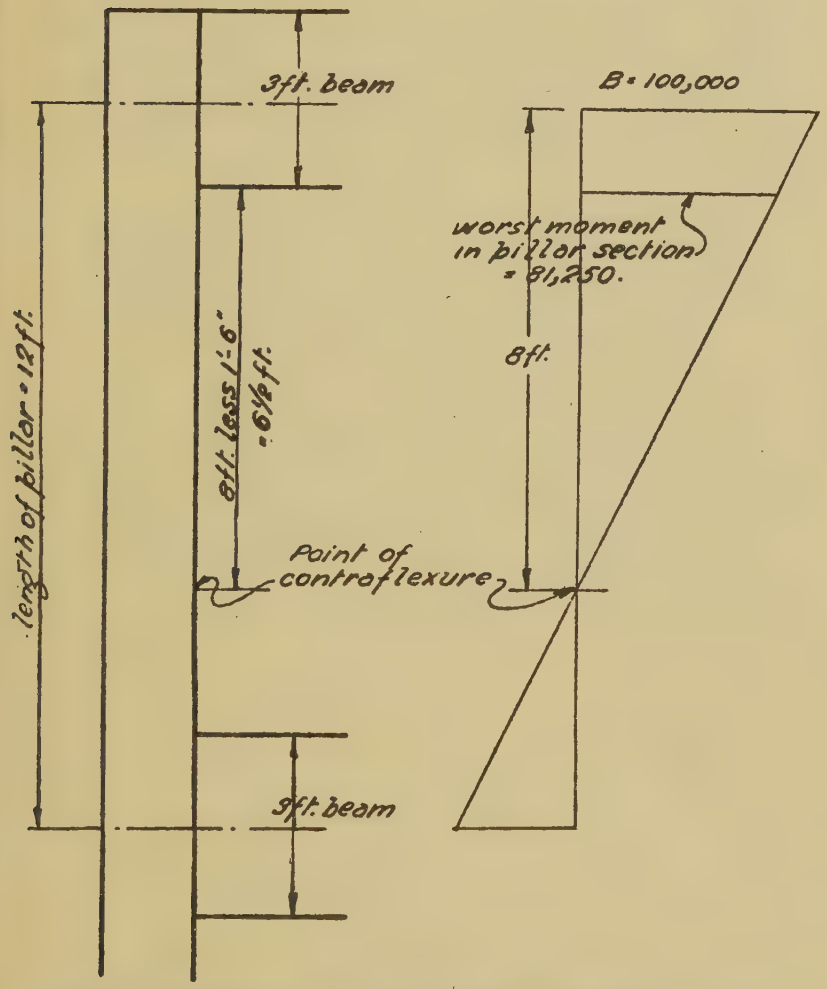


FIG. 28.—Showing reduction of Pillar Moment owing to depth of Beams or Haunches.

at the bottom of the beam, or if there is a haunch, at the bottom of the haunch.

Knowing the moment at the end, and the position of

the point of contraflexure, it is then a simple matter to calculate the reduced moment at this section.

For example, suppose a certain interior pillar with its lower end fixed (see Case II and Fig. 25) has a moment at its deflected end of 100,000 in. lb. (calculated from Table III, *R.C.S.E.*, p. 55).

Suppose now the beam producing this deflection is 3 ft. deep and the pillar is 12 ft. high (see Fig. 28). Then the position of the point of contraflexure is

$$\tfrac{2}{3}l = 8 \text{ ft.}$$

from the end. But the bottom of the beam is half of 3 ft., namely $1\frac{1}{2}$ ft., below the end of the pillar. Hence the worst moment in the pillar section is reduced to

$$100{,}000 \times \frac{(8 - 1\frac{1}{2})}{8} = 81{,}250 \text{ in. lb.}$$

This reduction is often important.

If we were considering an outside pillar and the case approximated to Case III, Fig. 26, we should take the point of contraflexure half-way down and the reduction of moment would be still more marked.

It will be seen that deep haunches may reduce the stresses in this way considerably.

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Printed wholly in England for the Muston Company,
By Lowe & Brydone, Printers, Ltd.
Park Street, Camden Town, London, N.W.1.

KV-166-420